On the Finiteness of Semigroups in Which $x^r = x$

Thomas C. Brown
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In this note we present a new proof of the following theorem (see [1]).

Theorem 1. *For each* r, $r \geq 2$, *every finitely generated semigroup in which* $x^r = x$ *holds identically is finite if and only if every finitely generated group of exponent* $r - 1$ *is finite.*

Notation. Throughout, r is fixed and S_k denotes a semigroup on k generators in which $x^r = x$. The elements of S_k are regarded as equivalence classes of words in k symbols $X_1, \ldots, X_k$. Upper case letters will denote words, and lower case letters the elements of the semigroup S_k; thus if the word W represents the element w of S_k, we write $W \in w \in S_k$, and also say that W

Proof of Theorem. One direction is trivial. For the other, we use induction on k; S_1 is obviously finite, and we suppose that S_{k-1} is finite. By Remark 1, there is a number m such that if $|X| = m$, where X is a minimal word of S_k, then X is complete in the k symbols $X_1, \ldots, X_k$.

Let W be any minimal word of S_k; then