On the canonical version of a theorem in Ramsey Theory

Tom C. Brown
Department of Mathematics
Simon Fraser University
Burnaby, BC Canada V5A 1S6
tbrown@sfu.ca

Citation data: T.C. Brown, *On the canonical version of a theorem in Ramsey theory*, Special Issue on Ramsey Theory, Combinatorics, Probability and Computing **12** (2003), 513–514.

Abstract

W5(,)0sF83 8.9664 Tf -84.763(8.960 g 0 G/F83 8.96688 0at0 g 0 ion)-2688constant0 g 0a,coin Ramseyy Theory

Proof of Theorem 4 The following facts follow directly from the definition of the colouring f.

Each aligned block of size 4^k has exactly 2^k colours, each appearing 2^k times.

Consecutive aligned blocks of size 4^k are either identical or have no colour in common.

Every set of 4^s consecutive non-negative integers has at least 2^s colours. Hence every set of 4^s consecutive aligned blocks of size 2^k has at least 2^s aligned blocks of size 2^k, no two of which have a common colour.

Now let $A \subset \omega$.

Choose s minimal so that A is contained in the union of two aligned blocks of size 4^s. (Two blocks are needed in case A contains $4^k - 1$ and 4^k.) Then $4^{s-1} < |A|gs(A)$, hence $|f(A)| \leq 2 \cdot 2^s < 2\sqrt{2|A|gs(A)}$.

Next, given $A \subset \omega$, choose k so that $2^{k-1} < gs(A) \leq 2^k$, and choose t minimal so that A is contained in the union of t consecutive aligned blocks of size 2^k. By the choice of k, A intersects each of these t blocks. Let $4^s \leq t < 4^{s+1}$. Since 2^s of these blocks have no colours in common, $2^s \leq |f(A)|$. Hence $|A| \leq t2^k < 4 \cdot 4^s \cdot 2 \cdot 2^{k-1} < 8|f(A)|^2 gs(A)$, or $\sqrt{|A|/8gs(A)} < |f(A)|$.

2 Acknowledgement

The author thanks the referee for substantial improvements in the bounds and the proofs.

References

[1] T.C. Brown, *On locally finite semigroups (in russian)*, Ukraine Math. J. **20** (1968), 732–738.

[2] ______, *An interesting combinatorial method in the theory of locally finite semigroups*, Pacific J. Math. **36** (1971), 285–289.

[3] A. de Luca and S. Stefano Varricchio, *Finiteness and regularity in semigroups and formal languages*, Springer-Verlag, Berlin Heidelberg New York, 1998.

[4] N. Hindman and D. Strauss, *Algebra in the stone-Čech compactification*, Walter de Gruyter, Berlin, New York, 1998.

[5] Gérard Lallement, *Semigroups and combinatorial applications*, Pure and Applied Mathematics. A Wiley-interscience Publication, John Wiley & Sons, New York-Chichester-Brisbane, 1979.

[6] H. Straubing, *The burnside problem for semigroups of matrices*, Combinatorics on Words, Progress and Perspectives, Academic Press, 1982, pp. 279–295.