Index

y bo s
1-form, 10
4-acceleration, 184
4-force, 115
4-momentum, 116
    total, 122
4-velocity, 114

A
acceleration
    4-acceleration, 184
    Newtonian, 74
action function or functional
    (see also Lagrangian), 594, 598
ADM action, 606
a ne parameter, 77, 79
alternating operation, antisymmetriza-
        tion, 27
angle field, 44
anisotropic fluid, 218–220, 276
    collapse, 424–431
anti-de Sitter space-time, 195, 644,
        663–664
anti-self-dual, 671
antisymmetric oriented tensor, 49
antisymmetric tensor, 28
antisymmetrization, 27
arc length parameter, 81
arc separation function, 85
arc separation parameter, 77
Arnowitt-Deser-Misner action integral,
        606
atlas, 3
    complete, 3
    maximal, 3

B
Betti number, 589
Bianchi type-I models, 448
Bianchi's di erential identities, 60
    complex valued, 532–533
    consequences of in Newman-Penrose

causality violation, 663
characteristic hypersurface, 623
characteristic matrix, 625
characteristic ordinary di erential equations, 612
characteristic polynomial equation, 177
characteristic surface, 619
charge-current, 125
charged dust, 125, 134, 226, 317
chart, 1
Christo el symbol, 57
  first kind, 57
  second kind, 57
closed form, 33
Codazzi-Mainardi equations, 100
commutator or Lie bracket, 40
comoving coordinate chart, 198
complete atlas, 3
complex conjugate coordinates, 621
complex electromagnetic potential, 356
complex gravo-electromagnetic potential, 356
complex null tetrad field, 485
complex potential, 331–332, 348
components
  coordinate, 21
  covariant, 11
  orthonormal, 47
  physical, 120
conformal group, 640
conformal mapping, 70
conformal tensor, 71
conformally flat domain, 71
conformally flat space-time, 149, 639–646
conformastat metric, 309
conformastationary metric, 358
conjugate holomorphic function, 622
conjugate points, 81
conjugate, contravariant, or inverse metric tensor, 44
constant curvature, space, 68, 73
continuity equation, 122, 126, 186, 232
continuum mechanics, 117
contraction operation, 21
contravariant index, 51
contravariant order, 18
coordinate chart or system
  Boyer-Lindquist, 334, 399
  Cartesian, 44, 68
  comoving, 198
  Doran, 415
  doubly-null, 130
  Eddington-Finkelstein, 371, 383
  Gaussian normal, 67, 202
  geodesic normal, 67, 102, 202
  Kerr-Schild, 414
  Kruskal-Szekeres, 379–380
  local, 1
  local Minkowskian, 153
  Minkowskian, 105
  normal or hypersurface orthogonal, 67
  orthogonal, 61
  Painlevé-Gullstrand, 370
  pseudo-Cartesian, 45, 68
  Regge-Wheeler tortoise, 371
  Riemann normal, 67
  Synge's doubly-null, 372
  Weyl's, 289
  Weyl-Lewis- Papapetrou (WLP), 330
coordinate components, 21
coordinate conditions, 165
cosmological constant, 163, 233, 435
cosmological principle, 434
cotangent vector field, 9
cotangent vector space, 9
Cotton-Schouten-York tensor, 309
covariant, 52
covariant components, 11
covariant derivative, 51
  of relative and oriented relative tensor fields, 63
covariant index, 51
covariant order, 18
covariant vector field, 9
critical energy density, 448
critical point, 593
curvature invariant, 63
curvature scalar, 63

curvature tensor, 55
curve
integral, 35
non-null, 81
null, 112
parameterized, 11
profile, 92
spacelike, 111
timelike, 112
curved domain, 64
Curzon-Chazy metric, 299
cusp, 16

D
dark energy, 461
de Sitter space-time, 431, 450, 644
deceleration function, 448
deceleration parameter, 448
deDonder gauge, 649
deformable solid, 220–222, 276
degree (system of p.d.e.s), 611
delta
Kronecker, 8
derivative
covariant, 51, 63
covariant directional, 52
directional, 7, 52
exterior, 31
Fermi, 147
gauge covariant, 561
Lie, 35–38
variational, 595, 599
derivative mapping, 22
determinate system, 166
di erentiable manifold, 3
di erential conservation of energy- momentum, 120
di erential form, 31
directional derivative, 7
domain
curved, 64
flat, 64
dominant energy condition, 181
Doran coordinate chart, 415
dot product or inner-product, 42
double Hodge-dual, 84
doubly-null coordinates, 130
dual vector space, 9
dubiosity, 82
dust, 122, 125, 134, 185, 186, 216, 226, 317, 435, 437, 438, 441, 448, 452, 470
collapse, 383–386

E
eccentricity, 245
Eddington-Finkelstein coordinates, 371, 383
Eguchi-Hansen gravitational instantons, 671
Einstein space, 69
Einstein static universe, 434
Einstein summation convention, 7
Einstein tensor, 63
Einstein's field equations, 164
complex-valued form, 532
Einstein-Hilbert Lagrangian density, 168
Einstein-Maxwell equations, 224, 263
Einstein-Maxwell-Klein-Gordon equations, 560
orthonormal form, 565
Einstein-Rosen bridge, 655
electrical charge density, 125
electro-vac universe, 263
electromagnetic duality-rotation, 233
electromagnetic field tensor, 33, 50
electromagnetic four-potential, 34, 225, 233, 504, 561
electromagneto-vac domain, 224
electrostatic potential, 319
elementary divisors, 632
elliptic p.d.e., 618
embedded manifold, 94
Emden equation, 575
energy conditions, 180
energy-momentum-stress tensor, 119
equation of state, 216
equilibrium of a deformable body, 221
equivalence principle, 139
ergosphere, 402
Ernst equation, 333

eternal black holes, 418
eternal white holes, 418
Euler equations for a perfect fluid, 232
Euler-Lagrange equations, 78, 159, 596
  classical mechanics, 596
  for particle in curved space-time, 157
  for particle in Schwarzschild space-time, 242, 243
  for scalar field, 599
event horizon, 367
exact form, 33
expansion scalar, 184
expansion tensor, 184
extended extrinsic curvature, 604
extended real numbers, 409
exterior derivative, 31
exterior product, 28
extrinsic curvature tensor, 89–104, 167
  extended, 604

F

F-L-R-W metrics (or Friedmann-Lemaître-Robertson-Walker), 436–471
F-W transport, 147
Fermi derivative, 147
Fermi-Walker transport, 147
fifth force, 643
fine-structure constant (electrodynamic), 583
fine-structure parameter (eigenvalue of charged scalar-wave gravitational condensate), 583
Finsler metric, 157
first contracted Bianchi's identities, 63
first curvature, 147
first fundamental form, 91
Flamm's paraboloid, 241
flat domain, 64
flat manifold, 67
Fock-deDonder gauge, 649
form
  1-form, 10
  closed, 33
  di erential, 31
  exact, 33
Frenet-Serret formulas, 83
  generalized, 82
frequency of a wave, 504
Friedmann-Lemaître-Robertson-Walker metrics (or F-L-R-W), 436–471

G

Gödel universe, 664–665
gauge covariant derivatives, 561
gauge transformation, 34
Gauss' equations, 100
Gauss' theorem, generalized, 65
Gauss-Bonnet term, 608
Gaussian curvature, 91
Gaussian normal coordinate chart, 67, 202
general covariance, 131
general relativity, 168
generalized D'Alembertian, 222
generalized Frenet-Serret formulas, 82
generalized Kronecker tensor, 29
generalized wave operator, 222
geodesic, 76
  non-null, 77
  null, 77
geodesic deviation equations, 80
geodesic equations, 77
  for dust, 186
  for particle in $\sqcap$-domain, 369
  for particle in Schwarzschild space-time, 242
  for the Euclidean plane $\mathbb{E}_2$, 78
geodesic normal coordinate chart, 67, 102, 202
geodesically complete manifold, 170
geometrized units, 164
geons, 591
Global Positioning System (G.P.S.), 336
gravitational constant, G, 66, 162
gravitational field equations, 163

gravitational instantons, 351, 667–671
gravitational mass, 138
gravitational redshift, 250
gravitational wave astronomy, 647
gravitational waves, 167, 647–653
gravitons, 649
gravo-electromagnetic potential, 356
group
  Bondi-Metzner-Sachs, 240
  conformal, 640
  Lie, 45
  Lorentz, 45, 109
  Möbius, 356
  Poincaré, 109
  symplectic, 173
group of motion, 73

H
H-K-S-D metric (or Hawking-Kloster-Som-Das), 670
Hamilton-Jacobi equation, 615
Hamiltonian
  for particle in a static gravitational potential, 326
  for particle in an electromagnetic field, 160
  relativistic, 158
  relativistic equations of motion, 160
  relativistic mechanics, 158
  super, 158
harmonic coordinate conditions, 174
harmonic function, 307, 622
harmonic gauge, 174, 649
Hausdor manifold, 1
Hawking-Kloster-Som-Das metric (or H-K-S-D), 670
heat flow vector, 232
Heaviside-Lorentz units, 123
Hessian, 616
Higgs fields, 471
Hilbert-Palatini approach for deriving field equations, 601
Hodge star operation, 50
Hodge-dual operation, 496
holomorphic function, 622
holonomic constraint, 158
homeomorphism, 1
homogeneity, 442
homogeneous p.d.e., 611
homogeneous state of the complex wave function, 570
Hopf's theorem, 307
Hubble function, 448
Hubble parameter, 448
hybrid tensor field, 96–98
hyperbolic p.d.e., 618
hypersurface, 367
  null, 131
hypersurface orthogonal coordinate, 67

I
I-S-L-D jump conditions, 167
identity tensor, 46
immersion, 94
incoherent dust, 185
index
  contravariant, 51
  covariant, 51
index lowering, 48
index raising, 48
inertial mass, 138
inertial observer, 113
inflationary era, 446
initial value problem, 207
inner-product or dot product, 42
instanton, 351, 667–671
instanton-horizon, 668
integrability conditions, 99
integral conservation, 120
integral curve, 35
intrinsic curvature
  extended, 604
intrinsic metric, 90
invariant eigenvalue problem, 91
invariant eigenvalues, 177
irrotational motions, 184
isomorphism, 501
  on tensor space, 48
isotropic sectional curvature, 441

isotropy, 441

Lorentz transformation, 45, 109
orthochronus, 192
Lorentz-Heaviside units, 123

M
magnetic potential, 315
manifold
constant curvature, 68, 73
di erentiable, 3
Einstein, 69
embedded, 94
flat, 67
geodesically complete, 170
Hausdor , 1
orientable, 3
paracompact, 1
pseudo-Riemannian, 64
Ricci flat, 70, 164
Riemannian, 64
Riemannian or pseudo-Riemannian, 56, 64
topological, 3
Maple(TM), 672
mass
gravitational, 138
inertial, 138
mass density, 118
mass-shell, 117
Mathematica(TM), 672
matrix
characteristic, 625
Lorentz, 133
nilpotent, 629
rank of, 18
unit, 8
matter-dominated era, 445
maximal atlas, 3
maximum-minimum principle, 307
Maxwell's equations, 34, 123
Maxwell-Lorentz equations, 124
mean curvature, 91
Meissner e ect, 571
metric
anti-de Sitter, 195, 644, 664
Bianchi type-I, 448
Brinkman-Robinson-Trautman, 514
components, 44
conformastat, 309
conformastationary, 358
conjugate, contravariant, or inverse, 44
Curzon-Chazy, 299
de Sitter, 431, 450, 644
Einstein static universe, 434
Finsler, 157
Friedmann-Lemaître-Robertson-Walker (or F-L-R-W), 436–471
Gödel, 664
Hawking-Kloster-Som-Das (or H-K-S-D), 670
Kasner, 212, 306, 310, 323
Kerr, 334–339, 399–417
higher dimensional generalization, 343
Kerr-Newman, 337, 415
Kottler, 252
Kruskal-Szekeres, 379–380
Lemaître, 280
Lorentz, 45, 64
positive definite, 43, 64
pseudo-Schwarzschild, 311
Reissner-Nordström-Je ery, 265, 394
Robinson-Trautman, 549
Schwarzschild, 239–251, 311, 350, 364–382
higher dimensional generalization, 253
Schwarzschild's interior, 263
signature, 43
Taub-NUT, 671
tensor, 42
Tolman-Bondi-Lemaître, 276
Vaidya, 284
Van Stockum, 665
warp-drive, 660
Weyl-Majumdar-Papapetrou-Das (or W-M-P-D), 319
wormhole, 656
metric equations, 321
metric tensor, 42

Minkowskian chart, 105
Minkowskian chronometry, 113
Monge surface, 102
monogenic forces, 156
most general solution, 609
multilinearity conditions, 18
Möbius group, 356

N
Newman-Penrose equations, 538, 545–547
Newtonian acceleration, 74
Newtonian approximation, 245
Newtonian gravitational constant, G, 66, 162
Newtonian gravitational potential, 66, 138, 142, 162, 169, 203, 280, 291–294, 299, 304, 319
Newtonian mass, 66
nilpotent matrix, 629
nilpotent operator, 33
no hair theorem, 412
non-degeneracy conditions, 11
non-linear eigenvalue problem for a theoretical fine-structure constant, 582
non-null di erentiable curve, 81
non-null geodesic, 77
norm or length, 43
normal coordinate, 67
normal forms, 619
normal or canonical forms, 510
null cone, 108
null curve, 112
null electromagnetic field, 231, 503
null energy condition, 180
null hypersurface, 131
null vectors, 106
numerical tensor, 29, 111
NUT potential, 669

O
Oppenheimer-Volko equation, 261
order (system of p.d.e.s), 611
order of a tensor, 18
orientable manifold, 3
oriented relative tensor field, 26
oriented tensor field, 50
oriented volume, 65
orthochronus Lorentz transformation, 192
orthogonal coordinate chart, 61
orthogonal vector fields, 42
orthonormal basis, 42
orthonormal components, 47
outer ergosurface, 402
outer product, 19
overdetermined system, 166

P
Painlevé-Gullstrand coordinate chart, 370
Palatini approach for deriving field equations, 601
Papapetrou-Ehlers class, 350
parabola, semi-cubical, 16
parabolic p.d.e., 618
paracompact manifold, 1
parallelly propagated, 74
parameterized curve, 11
parametric surface, 89
parametrized hypersurface, 94
Penrose-Carter compactification, 409
perfect fluid, 180, 216–218
perihelion, 245
perihelion shift, 246
permutation symbol, 30

complex electromagnetic, 356
complex gravo-electromagnetic potential, 356
electromagnetic four, 34, 225, 233, 504, 561
electrostatic, 319
equation, 321, 323
function, 307
gravo-electromagnetic potential, 356
magnetic, 315
Newtonian gravitational, 66, 138, 142, 162, 169, 203, 280, 291–294, 299, 304, 319
NUT, 669
scalar field, 462, 464
twist, 347, 353, 356
potential equation, 321, 323
potential function, 307
pressure, 180
principal curvature, 147
principal pressures, 182
principal tensions, 182
principle of equivalence, 139
Proca equation, 571
profile curve, 92
projection tensor, 183
proper time, 112
pseudo-angle, 44
pseudo-Cartesian chart, 45, 68

quintessence, 461–466
scalar field potential, 462, 464
Schwarzschild metric, 239–251, 311, 350, 364–382
higher dimensional generalization, 253
Schwarzschild radius, 240, 364
Schwarzschild's interior solution, 263
second contracted Bianchi's identities, 63
second fundamental form, 91
Segre characteristic, 632–634
of energy-momentum-stress tensors, 177–180
of Ricci tensor, 516
self-dual, 670
self-duality, 497
semi-cubical parabola, 16
semi-linear p.d.e., 611
semiclassical gravity, 561
separation of a vector field, 42
shear tensor, 184
shock waves, 610
signature of the metric, 43
singularity theorems, 432–433, 449
solid particle, 228
space of constant curvature, 68, 73
space-time foam, 654, 655
spacelike curve, 111
spacelike vectors, 106
spin coe cients, 538
spinor algebra, 547
spinor analysis, 547
spinor fields, 111
stationary limit surface, 402
Stokes' theorem, generalized, 34
stress density, 118
stress tensor field, 119
string theory, 468, 472, 479, 560
strong energy condition, 181
summation convention, 7
surface
Monge, 102
parametric, 89
ruled, 85, 97
surface of revolution, 92
Sylvester's law, 623
symmetric tensor, 27
symmetrization operation, 27
symmetrized curvature tensor, 155, 172
symplectic group, 173
Synge's doubly-null coordinates, 372
Synge's junction conditions, 166
Synge's world-function, 86

T-domain, 280
tachyon condensation, 468
tachyonic (or Born-Infeld) scalar field, 467–471
tangent vector, 6, 7
tangent vector space, 7
Taub-NUT metric, 671
tensor
antisymmetric, 28
canonical energy-momentum-stress, 119
conformal, 71
conjugate, contravariant, or inverse metric, 44
Cotton-Schouten-York, 309
curvature
Riemann, 55
extended extrinsic, 604
extrinsic, 89–104, 167
Ricci, 62
Riemann-Christo el, 59, 62
symmetrized, 155, 172
density, 26
Einstein, 63
electromagnetic field, 33, 50
energy-momentum-stress, 119
expansion, 184
extended extrinsic curvature, 604
extrinsic curvature, 89–104, 167
field, 20
generalized Kronecker, 29
hybrid field, 96–98
identity, 46
Killing, 414
Levi-Civita, 49

metric, 42
numerical, 29, 111
order of, 18
oriented, 26, 50, 63
oriented antisymmetric, 49
polarization, 651
projection, 183
rank of, 18
relative, 26, 63
Ricci, 62
Riemann, 55
Riemann-Christo el, 59, 62
shear, 184
stress, 119
symmetric, 27
symmetrized curvature, 155, 172
torsion, 54
totally antisymmetric, 27, 49
trace-reversed perturbation, 648
vorticity, 184
Weyl conformal, 71
Weyl's projective, 71
tensor density field, 26
tensor field, 20
tensor product, 19
tetrad, 106, 481
complex null, 485
Theorema Egregium, 101
tidal forces, 143
time machine, 662–666