

# Contents

Preface . . . . .	vii
List of Figures . . . . .	xiv
List of Tables . . . . .	xviii
<b>1 Tensor Analysis on Differentiable Manifolds . . . . .</b>	<b>1</b>
1.1 Differentiable Manifolds . . . . .	1
1.2 Tensor Fields over Differentiable Manifolds . . . . .	18
1.3 Riemannian and Pseudo-Riemannian Manifolds . . . . .	42
1.4 Extrinsic Curvature . . . . .	89
<b>2 The Pseudo-Riemannian Space-Time Manifold <math>M_4</math> . . . . .</b>	<b>105</b>
2.1 Review of the Special Theory of Relativity . . . . .	105
2.2 Curved Space-Time and Gravitation . . . . .	138
2.3 General Properties of $T_{ij}$ . . . . .	177
2.4 Solution Strategies, Classification, and Initial-Value Problems . . . . .	200
2.5 Fluids, Deformable Solids, and Electromagnetic Fields . . . . .	216
<b>3 Spherically Symmetric Space-Time Domains . . . . .</b>	<b>237</b>
3.1 Schwarzschild Solution . . . . .	237
3.2 Spherically Symmetric Static Interior Solutions . . . . .	256
3.3 Non-static, Spherically Symmetric Solutions . . . . .	269

6.3	Five-dimensional Cosmological Models . . . . .	471
<b>7</b>	<b>Algebraic Classification of Field Equations . . . . .</b>	<b>481</b>
7.1	The Petrov Classification of the Curvature Tensor . . . . .	481
7.2	Newman-Penrose Equations . . . . .	524
<b>8</b>	<b>The Coupled Einstein-Maxwell-Klein-Gordon Equations . . . . .</b>	<b>560</b>
8.1	The General E-M-K-G Field Equations . . . . .	560
8.2	Static Space-time Domains and the E-M-K-G Equations . . . . .	566
8.3	Spherical Symmetry and a Non-linear Eigenvalue Problem for a Theoretical Fine-Structure Constant . . . . .	575
<b>Appendix-1 Variational Derivation of Differential Equations . . . . .</b>		<b>593</b>
<b>Appendix-2 Partial Differential Equations . . . . .</b>		<b>609</b>
<b>Appendix-3 Canonical Forms of Matrices . . . . .</b>		<b>628</b>
<b>Appendix-4 Conformally Flat Space-Times and “the Fifth Force” . . . . .</b>		<b>639</b>
<b>Appendix-5 Linearized Theory and Gravitational Waves . . . . .</b>		<b>647</b>
<b>Appendix-6 Exotic Solutions: Wormholes, Warp-Drives, and Time Machines . . . . .</b>		<b>654</b>
<b>Appendix-7 Gravitational Instantons . . . . .</b>		<b>667</b>
<b>Appendix-8 Computational Symbolic Algebra Calculations . . . . .</b>		<b>672</b>
<b>References . . . . .</b>		<b>680</b>
<b>Index of Symbols . . . . .</b>		<b>690</b>
<b>Index . . . . .</b>		<b>698</b>