Stochastic Modelling and Comparison of Two Pension Plans

by

Zetong Li

B.Sc., Tsinghua University, 2014

Project Submitted in Partial Ful llment of the

Approval

Name: Zetong Li

Degree: Master of Science (Actuarial Science)

Title: Stochastic Modelling and Comparison of Two Pension Plans

Examining Committee: Chair: Tim Swartz
Professor

Ms. Barbara Sanders
Senior Supervisor
Assistant Professor

Dr. Gary Parker
Co-supervisor
Associate Professor

Dr. Yi Lu
Internal Examiner
Associate Professor

Date Defended: 19 April 2017

Abstract

In this project, we simulate the operation of a stylized jointly sponsored pension plan (JSPP) and a stylized defined contribution (DC) plan with identical contribution patterns using a vector autoregressive model for key economic variables. The performance of the two plans is evaluated by comparing the distribution of pension ratios for a specific cohort of new entrants. We find that the DC plan outperforms the JSPP in terms of expected pension ratio, and experiences only a moderate degree of downside risk. This downside risk is not enough to outweigh the upside potential even for a relatively risk-averse member, as reflected in the expected discounted utility of benefits under the two plans. Under more sophisticated rate stabilization techniques, the probability that the DC plan outperforms the JSPP increases. When the bond yield and stock return processes begin from values far above their long-term means (not far below, as is the case today), the DC plan is projected to outperform the JSPP even more frequently, because the higher required contributions accrue to the advantage of the individual member only, instead of also financing benefits for others.

Keywords: Pension Plan Comparison; Jointly Sponsored Pension Plan; Stochastic Simu-

Acknowledgements

I would like to thank my supervisor, Ms. Barbara Sanders, for her comprehensive guidance on this project. She has been very patient, generous and encouraging with me. I truly appreciate her dedication of time and e orts, without which this project could not be accomplished smoothly.

I also owe my great gratitude to Dr. Gary Parker, for o ering me the opportunity to study in this program and supporting me throughout my studies. His wisdom, kindness and good humor will inspire me in my future study and career development.

I would also like to thank the committee member, Dr. Yi Lu, who has made valuable comments and helped re ne the report. Thanks are due as well to the faculty and sta in the department of Statistics and Actuarial Science. These last three years have brought me far more than I could have imagined.

Last but not least, I am grateful to the fellow graduate students and other friends for making this journey a memorable experience. As always, I am grateful to my parents, with whose endless love and unconditional support I could be where I am today.

Contents

List of Figures

Chapter 1

Introduction

1.1 Background and Motivation

Over the past few decades, occupational pension coverage has shifted from traditional de- ned bene t (DB) plans towards de ned contribution (DC) pension plans in many countries. Factors contributing to this shift include increasing labor mobility, sustained problems with DB underfunding, and regulatory changes (Broadbent et al., 2006). For voluntary employer pensions, the shift towards DC pension plans was most pronounced in the U.S.. In Australia, where occupational pensions are mandatory, DC plans now cover most of the workforce. Canada was less a ected by the shift, with most large public sector plans remaining DB. However, even in the public sector "some interest groups are pressing hard to convert their pension plans from DB to DC" (Brown and McInnes, 2014, p. 3).

DC plans provide portability and investment exibility during the accumulation phase, but leave members with signi cant uncertainties in the decumulation phase. In most modern DC plans, members are responsible for making their own investment decisions. Whether these plans can provide an adequate income in retirement is thus linked to members' nancial literacy, which is generally lacking. According to Brown and McInnes (2014), "while successful at vastly increasing DC pension coverage, the Australian program has been less successful at reducing poverty in seniors and displays on a large scale the problems associated with individually controlled savings plans" (p. 20). In the U.S., average pensions from DC plans fall short of the average bene ts payable under DB plans.

Financial markets have also changed greatly. DC plan members with large allocations to equities may have bene ted from the sharp rise in equity prices in the mid 1990s, while those who joined more recently have seen smaller rewards and greater volatility. A decline in long-term interest rates has also reduced the guaranteed income that the DC account balance could secure upon retirement.

DC plan for faculty members for over 40 years. Now that the plan is mature and members are beginning to retire in significant numbers, they are finding that the benefits provided are inadequate. To provide faculty members with more predictable benefits at a better price, the SFU Faculty Association is considering switching from the current DC plan to the B.C. College Pension Plan, a jointly sponsored pension plan (JSPP). According to Kristjanson and Darrach (2012), a "JSPP is a contributory, defined benefit pension plan in which all contributing stakeholders and plan members have decision making and funding responsibility" (p. 2). JSPPs are unique in their funding structures, where employees and employers both have potentially unlimited risk and share it equally. Most JSPPs are governed by boards of trustees or directors, and are frequently used by hospital associations and public sector unions in Canada.

As suggested in the report prepared by PBI Actuarial Consultants Ltd. (2015), the College Pension Plan is able to mitigate risks, including investment risk, inflation risk and longevity risk, and provide a more certain retirement income. It also has the advantage of low management and administration expenses. The report contains a number of comparisons that approximate the retirement income members could receive depending on their current age and retirement age under various return scenarios. In most scenarios, the College Pension Plan tends to outperform the current plan. Most members, except for those with strong financial literacy skills, would be better off in the College Pension Plan that removes much of the investment risk and provides a predictable pension benefit.

One limitation of the PBI report is that all assumptions are deterministic. Without accounting for the volatility in future salary increases, inflation, investment returns and annuity purchase rates, the comparison lacks an important dimension: risk. This affects both options: in terms of benefit volatility under the current DC plan, and contribution volatility under the College Pension Plan. Our motivation is to extend the work done by PBI to a stochastic context and compare the value of the two plan options to a particular cohort of new entrants. We focus on new entrants exclusively because, if SFU were to join the College Pension Plan, enrollment would be mandatory for new employees, while existing faculty members would have the option to stay in the current DC plan.

In order to stochastically simulate the operation of the two plans, we need an asset model to generate future economic scenarios. In dealing with economic variables, often the value of one variable is not only related to its predecessors in time, but also depends on past values of other variables. Consequently, we choose a vector autoregressive (VAR) model, which assumes a linear relationship between multiple economic variables, and predicts future values based on linear functions of past observations. In a VAR model, there are n state variables and n equations to express the relationship between each variable and its own lagged values, as well as current and past values of the remaining $n-1$ variables.

A reasonable performance criterion to evaluate alternative pension plans with comparable contribution levels is the pension ratio: the ratio of benefits under one plan to benefits under the other. We use the value-at-risk (VaR) of the pension ratio to assess how often the SFU plan can deliver the same or better benefits as the College Pension Plan. In addition, we use expected discounted utility to perform a welfare comparison of benefits under the two schemes. Expected utility theory has been used as a major paradigm in decision making problems (Schoemaker, 1982). It serves as the second performance criterion in our study, and provides additional evidence about whether the representative cohort would benefit from joining the College Pension Plan.

1.2 Literature Review

Most of the actuarial literature relating to quantitative comparisons of alternative pension plans focus on pure DB and DC plans. Samwick and Skinner (1998) used a detailed survey of pension formulas in the Survey of Consumer Finance to estimate the average pension benefit for a sample of both plans. They found that DC plans could strengthen the financial security of retirees, and their conclusion was robust to a number of specifications. However, the paper was written in a period when stocks provided high returns. Also, some key variables used in the analysis, such as annuity purchase rate, were fixed and did not anticipate the downward trend in long term interest rates. Their findings might be very different today.

Blake, Cairns and Dowd (2001) investigated a range of stochastic asset return models and asset allocation strategies, to estimate the distribution of future pension ratios (i.e., ratios of DC pension to DB pension). They explored the dynamics of interest rates, earnings, unemployment and asset allocation. The application of the well-established risk measure, value-at-risk, provided a simple and practical-to-implement methodology to evaluate alternative pension plans. The conclusion was that DC plans can be extremely risky relative to a DB benchmark. Value-at-risk estimates were most sensitive to the choice of asset-allocation strategy, and less sensitive to the choice of asset model. Asset models used in this paper included the Wilkie (1986) model, which was the first stochastic model for use by actuaries that incorporated a cascade structure, where each variable depends only on prior values of that variable and the values of variables that lie above them on the cascade structure. Under this structure, once a variable is appropriately calibrated, the calibration of subsequent variables lower on the cascade structure will have no impact on the previously calibrated variables.

Blake, Cairns and Dowd (2003) compared alternative decumulation strategies, including a conventional life annuity, an equity-linked annuity, and an equity-linked income distribution programme. To measure the performance of different strategies, they calculated the plan member's expected discounted lifetime utility. This framework captured an individual's attitude towards risk, and allowed the authors to optimize asset portfolios by maximizing the utility function. They concluded that the optimal choice of distribution programme was fairly insensitive to a member's risk-aversion level, but was greatly a ected by equity proportions.

to be more efficient and sustainable forms of risk sharing, compared to traditional DB or DC plans. However, the choice of a specific pension arrangement depends on the preferences of plan members, and in particular their degree of risk aversion and their ability to commit

Chapter 2

Economic Scenario Generator

2.1 The VAR Model

As in Heidelberg (2005), we model the return dynamics by a rst-order VAR model,

$$z_{t+1} = \quad z_t + P \quad _{t+1} \tag{2.1}$$

where z_t is a $(5 \quad 1)$ vector of centered state variables and $_{t+1} \overset{i:i:d}{\quad} N(0; I)$ is a $(5 \quad 1)$ vector of innovations. More precisely,

$$z_t = x_t \tag{2.2}$$

where x_t is the $(5 \quad 1)$ vector of original state variables and   is the vector of their historical means. By subtracting , we rule out the intercept term in the VAR model, as well as estimation inaccuracy on it.   and P are both $(5 \quad 5)$ matrices.   contains the autoregressive coe cients of the VAR model and P is the Cholesky decomposition of the covariance matrix   for residuals. In other words, P is a lower triangular matrix and satis es

$$PP^T = \quad : \tag{2.3}$$

2.2 Data and Parameter Estimation

The state variables that enter the VAR model include price in ation, 1-month interest rate, 10-year zero-coupon bond rate, stock return from the Toronto Stock Exchange (TSX) index, and stock return from the Standard & Poor's 500 (S&P 500) index. We express the S&P 500 index in Canadian dollars to rule out uctuations in currency exchange rates, Bank of Canada adopted an in ation-control target in 1991, which "aims to keep total CPI in ation at the 2 per cent midpoint of a target range of 1 to 3 per cent over the medium term" (Bank of Canada, n.d.). Data is available from the following sources:

Values of the consumer price index with base year 2002, considering all of Canada and not excluding any items are retrieved from CANSIM table 326-0020. The force of monthly in ation in month t is de ned as:

$$\tilde{\ }_t = \ln \frac{CPI_t}{CPI_{t\ 1}}; \tag{2.4}$$

where CPI_t is the value of the index at the end of month t.

The yield on 1-month Canadian treasury bills in month t (i_t^1) is retrieved from CANSIM table 176-0043, where it is quoted as an annual e ective rate. The corresponding monthly force of interest on 1-month treasury bills is de ned as:

$$y_t^1 = \frac{1}{12} \ln (1 + i_t^1): \tag{2.5}$$

The yield curves for zero-coupon bonds with terms to maturity ranging from 3 months to 30 years are available from the Bank of Canada on a daily basis. The 10-year bond yield observed on the rst trading day of month t (i_t^{120}) is used as a proxy for the long-term interest rate. The 10-year bond yield at the beginning of month t, expressed as the force of interest, is de ned as:

y_t 1rdalSJ/F22 7.9701 Tf -0.3921-7.201 Td [(t)]TJ/F18 10.9091 Tf 46.62422.697 Td [(=)]TJ 15.438)i (

TF83(ar)-2680Canadian esquty eetrurnsthlye&P/TSX-3680Ccomonsit fromyANSIMy the ylrgettartho-2051tSo hk-4152oExhcr-3152odivided yieldie tTed as:
yontrntuou-1(suy)-336(c)omonudexd-336(con)27(thly)-336(thtal)-3116eetrurn-3117onth

i+1]TJ/F22 7.9701 Tf 6.3614

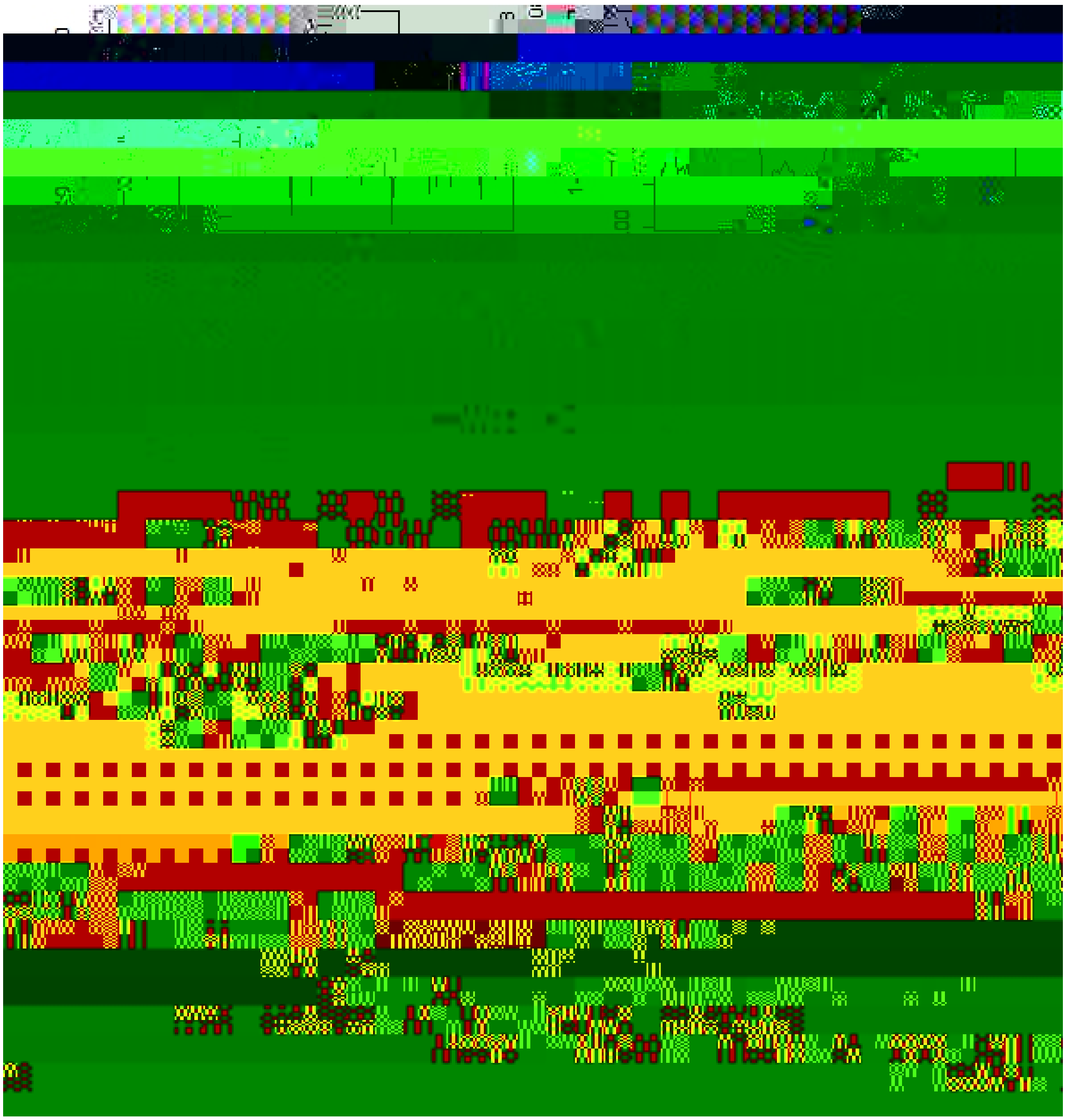
continuously compounded monthly total return on U.S. equities during month t as:

$$\tilde{}_t^U = \ln\left[\frac{SEI_t}{SEI_{t-1}} + \frac{SDY_t}{12}\right]; \tag{2.8}$$

where SEI_t is the value of the S&P 500 equity index at the end of month t and SDY_t is the annual dividend yield on the S&P 500 in respect of month t.

Figure 2.1 shows historical data for these five variables converted to annual scale. While bond yields depend greatly on past values and suggest strong auto-correlation, there is no significant pattern for inflation rates and equity returns. Current interest rates are at a historically low level. Summary statistics of historical data can be found in Table 2.1 panel (a).

Figure 2.1: Historical data of the VAR model



Panel (b) and (c) in Table 2.1 show estimates of and P with p-values in parentheses, obtained by using the R package vars. Inflation appears to be weakly related to Canadian equity returns besides its own lag, but the corresponding R^2 (0.0551) is very low. Stock returns have even less relationship to lagged variables; with an R^2 of 0.0335 and 0.0027, and no significant autocorrelations, these returns more or less follow white noise processes with highly correlated innovations. Panel (c) also confirms that stock returns have the highest volatility. Interest rates are mostly explained by their own lagged values, and the volatility of short term yield is higher than the volatility of the long-term yield.

When we fit the VAR model, an important assumption is that the process is stationary, that is, its statistical properties such as mean and autocovariances are fixed and do not change over time. Stationarity is crucial for being able to describe the stochastic behavior by the simple VAR model and to estimate the parameters. As introduced in Heidelberg (2005), the stationarity condition for a VAR(1) model requires all eigenvalues of have modulus less than 1. Here the absolute values of the eigenvalues are 0.9892, 0.9317, 0.1453, 0.1453 and 0.0266. Since they are all smaller then one, the stationarity condition is satisfied.

Table 2.1: Summary statistics and VAR estimation results

a) Summary statistics	$\tilde{}_t$	y_t^1	y_t^{120}	$\tilde{}_t^C$	$\tilde{}_t^U$	
	0.0015	0.0026	0.0040	0.0066	0.0076	
	0.0034	0.0017	0.0017	0.0420	0.0365	
b) VAR estimates ( )	$\tilde{}_t$	y_t^1	y_t^{120}	$\tilde{}_t^C$	$\tilde{}_t^U$	R^2
$\tilde{}_{t+1}$	0.1415	0.1116	-0.1101	0.0157	-0.0107	0.0551
	(0.0154)	(0.6183)	(0.6206)	(0.0066)	(0.1017)	
y_{t+1}^1	-0.0005	0.9510	0.0300	-0.0001	0.0001	0.9724
	(0.9200)	(0.0000)	(0.1170)	(0.8730)	(0.8750)	
y_{t+1}^{120}	0.0009	0.0239	0.9707	-0.0002	-0.0001	0.9876
	(0.7795)	(0.0675)	(0.0000)	(0.5683)	(0.7798)	
$\tilde{}_{t+1}^C$	-0.1494	-2.9637	3.2773	0.1226	0.0725	0.0335
	(0.8383)	(0.2938)	(0.2427)	(0.0911)	(0.3809)	
$\tilde{}_{t+1}^U$	-0.0491	-0.7141	1.4017	-0.0038	0.0328	0.0027
	(0.9390)	(0.7740)	(0.57)1500.7740)			

b) Vcess(pro306(AR)-334(4stimates)-3 Td [(~)]TJ/F291.788091 Tf 21.095 0 Td [(P)]TJ/F18.51.9091 Tf 7.879 0 Td [()]TJ ET q 1

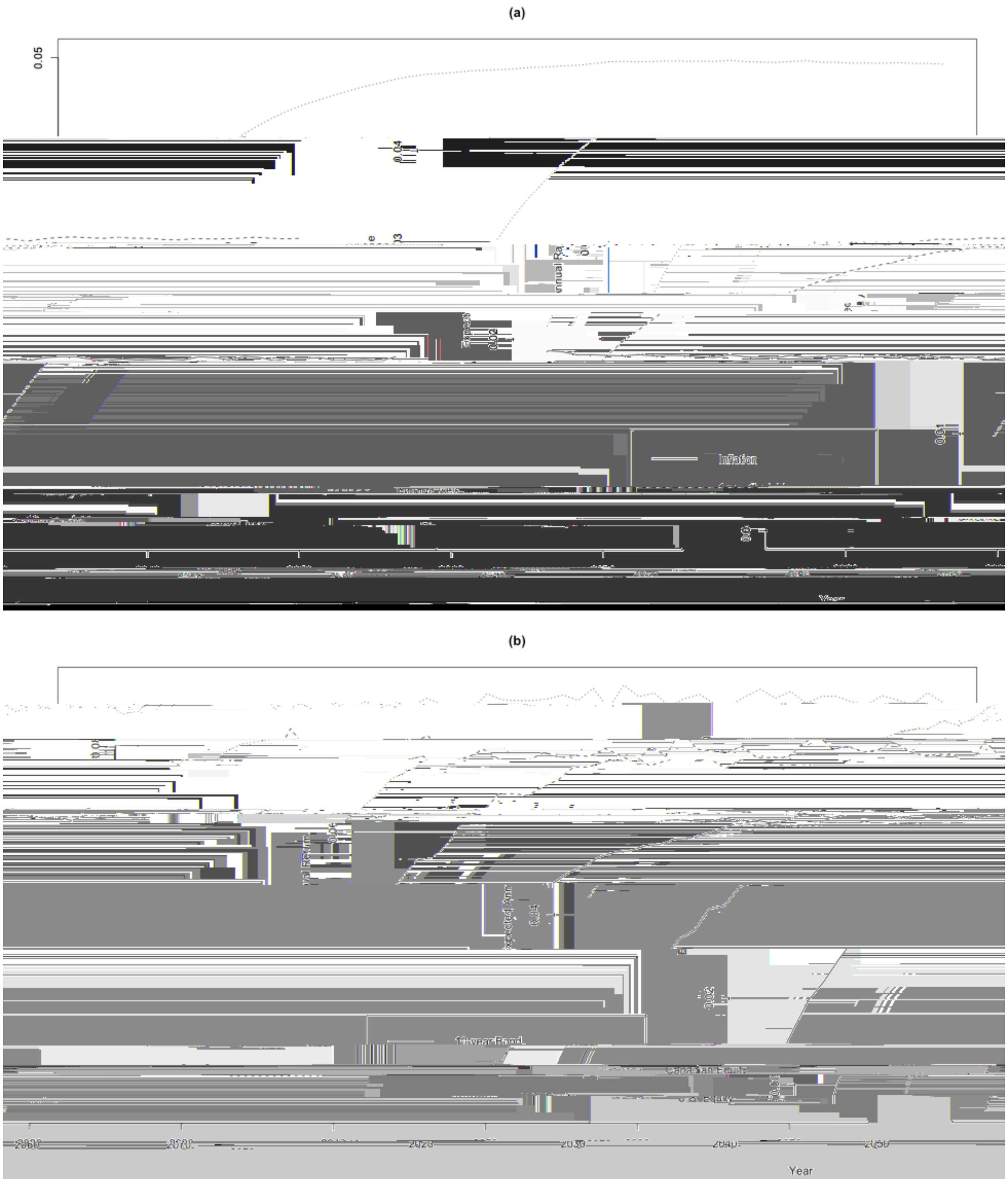
2.3 Simulation Results

The continuously compounded 10-year spot rate applicable at the end of year t (month $12t$), expressed as an annual rate, is denoted by y_t^{120} and is obtained directly from the simulations:

$$y_t^{120} = 12 \cdot y_{12t}^{120}. \tag{2.13}$$

Let $P^{(n)}$

Figure 2.2: Simulation results



Chapter 3

Simulation of Stylized Jointly Sponsored Pension Plan (JSPP)

In this chapter, we rst introduce the College Pension Plan. Our goal is to simulate the operation of a plan similar to the College Pension Plan with certain simpli cations. General assumptions and notation are presented in section (3.2), which are followed by features of the stylized JSPP in section (3.3). To investigate the impact of rate stabilization techniques, we describe two alternative designs in section (3.4).

3.1 BC College Pension Plan

British Columbia's public sector pension plans (BC plans) include the College Pension Plan, the Municipal Pension Plan, the Public Service Pension Plan and Teacher's Pension Plans (Municipal Pension Plan, n.d.). They are pre-funded so each generation pays in advance for its own pension bene ts. Costs and risks are shared between employees and employers. A basic element of each of these plans is that guaranteed pensions are based on a DB formula using the member's pensionable service and salary. Another element is in ation protection. This is not a guaranteed bene t and is provided based on the availability of funds. Contributions may change depending on the funded status of the plan. Each plan uses the BC Investment Management Corporation as its investment agent, which provides sophisticated and low-cost investment management of the funds. The total cost of investment management and pension administration for the plans is about one quarter of one per cent.

The College Pension Plan, designed almost 50 year ago, is by far the smallest of the four BC's public sector pension plans. It maintains retirement bene ts for around 25,000 senior administrators and faculty providing educational services at 23 BC colleges and universities. In 2000, the College Pension Plan shifted from government sponsorship to joint sponsorship and trusteeship. The plan is funded by employee and employer contributions, and under the new model risks are shared equally by the two parties. The joint trust agreements require that contribution rates and bene ts be reviewed triennially based on an actuarial valuation. The features described below are from College Pension Plan 2015 Annual Report (College Pension Plan [CPP], 2016a), College Pension Plan Funding Policy (CPP, 2016b), and College Pension Plan Statement of Investment Policies and Procedures (CPP, 2016c).

1. Demographic Pro le:
   Plan membership consists of 13,807 active members who are currently contributing (54% of the membership); 5,170 inactive plan members who have terminated their employment but left their bene ts in the plan (20% of the membership) and 6,453 retired plan members who are receiving a pension, including a survivor or disability pension (26% of the membership).

2. Contributions:
   Both plan members and employers pay contributions to fund future pension bene- ts; plan members contribute through automatic deductions from their employment earnings. A portion of these contributions goes to the basic account, which covers members' basic pensions; another portion goes to the in ation adjustment account, which covers cost-of-living adjustments (COLAs). Table 3.1 is a summary of contribution rates as a percentage of salaries from the College Pension Plan 2015 Annual Report.

Table 3.1: College Pension Plan contribution rates

E ective Date	On salary up to YMPE [1]		On salary over YMPE	
	Member	Employer	Member	Employer
January 1, 2016	9:86%	9:96%	9:86%	9:96%
September 1, 2013	9:60%	9:70%	10:35%	10:45%

[1] YMPE = Year's Maximum Pensionable Earnings, the maximum earnings on which Canada Pension Plan contributions are made.

3. Asset Allocation and Investments:
   When members retire, their pension is funded by their own contributions, their employers' contributions and investment returns. Based on current assumptions, approximately 30 cents of every dollar a retired member receives come from contributions they made and their employer made; the remaining 70 cents come from investment returns. To achieve the objective of meeting the pension bene ts promise, the Board has adopted the long term asset mix and allowable ranges as shown in Table 3.2. Diversifying investments is a sound way to balance investment risk while generating returns, especially in a global economy where turbulence is not uncommon. In the last three years, College Pension Plan investment portfolio earned 7.3% net of fees for

the scal year 2014/15, 17.5% for 2013/14, and 10.3% for 2012/13, which all exceeded the market benchmark.

Table 3.2: College Pension Plan asset mix: allowable ranges and long term policy

Asset Class	Allowable Range (%)		Long Term Policy
	Minimum	Maximum	Asset Mix(%)
Short Term	0	10	2
Mortgages	0	10	5
Nominal Bonds	5	17	10
Real Return Bonds	0	10	5
Fixed Income Sub-total	12	35	22
Canadian Equities	8	18	13
Global Equities	17	29	22
Emerging Markets	4	14	9
Public Equity Sub-total	35	55	44
Real Estate[1]	11	23	18
Real Estate Sub-total	11	23	18
Private Placements[1,2]	1	11	6
Infrastructure and Renewable Resources[1,2]	5	15	10
Sub-total	7	22	16
Other[3]	0	5	5

[1] Due to the illiquid nature of these assets, the upper limit may be exceeded on a temporary basis.
[2] Private Placements may be either debt or equity.
[3] Other includes strategies or investments speci cally approved by the Board that do not correspond to the listed asset classes.

4. Basic Pension:

   The College Pension Plan provides members or their bene ciaries with a basic lifetime pension bene t based on highest average salary and years of service. Here "highest average salary" means the average annual salary earned by a member during the 5 years of pensionable service in which the salaries were highest. Normal retirement age is 65 for all members. The unreduced guaranteed bene t is calculated in the form of a single life annuity guaranteed for 10 years:

   2% ve-year highest average salary total pensionable service (years).

5. Cost-of-living Adjustments (COLA):

   Cost-of-living Adjustments to pensions in pay are managed through a separate in a-tion adjustment account. Future increases are not guaranteed; however, once granted, COLA becomes part of the members' basic lifetime pension. On January 1, 2015, retired members received a COLA of 1.83 per cent. The COLA cannot exceed the change in the consumer price index or the in ation adjustment cap set every three

3.2 General Assumptions and Notation

In our model, we make some important assumptions.

1. Contributions are received and bene ts are paid at the beginning of the year.

2. Expenses related to management and administration are ignored.

3. The source of contributions (employer vs. member) is irrelevant. All contributions are considered together.

Next, we introduce some notation. We let:

e be the entry age,

r be the retirement age,

l be the life expectancy at time of retirement,

r_t^P be the annual portfolio return during the period $[t \quad 1; t)$,

s_t be the actual salary increase rate at time t,

$a_{\overline{h}|i}$ be the present value of ane ent where $FAS_{x;t;k}$ is the average annual salary earned by a member during the last 5 years of pensionable service:

$$FAS_{x;t;k} = \frac{1}{5}\Big(Sal_{r-1;r-e+3k-1;k} + Sal_{r-2;r-e+3k-2;k} + Sal_{r-3;r-e+3k-3;k} + Sal_{r-4;r-e+3k-4;k} + Sal_{r-5;r-e+3k-5;k}\Big); \tag{3.4}$$

and the age at entry, e, is equal to $x - (t - 3k)$.

Note that we have replaced highest average salary with final average salary. This is a reasonable simplification, since 97% of the simulated salary increase rates are positive and the minimum value is no lower than -4%, so final average salary is almost identical to the highest average salary used by the College Pension Plan. Total benefit payments made from

3.3.2 Assets

The asset allocation is designed to reflect the characteristics of the College Pension Plan using the state variables available in the VAR model. As Table 3.2 suggests, real estate, private placements, infrastructure and renewable resources are considered illiquid, thus we only include fixed income and public equity in the asset portfolio. Note that we use an index to model equity returns while the actual fund adopts active management which has outperformed the index. In this case, our assumption is more conservative.

Since short-term bonds only make up a small percentage of the portfolio, we let the entire fixed income allocation of the stylized JSPP consist of 10-year bonds only, with a weight of 35% of the total portfolio ($\frac{22\%}{22\%+44\%}$). Canadian equities are

3.3.3 Valuation Assumptions

The assumptions we need for each valuation are future valuation rates and future salary increases. These vary at each time point under each scenario. We set y_t^V, the funding valuation rate at time t, as the expected return on assets ($EROA_t$) subject to some restrictions. We assume that $EROA_t$ can be constructed by adding a risk premium to the long-term interest rate. From Table 2.1 panel (a), we find that the historical 7.15 -16 set where

$B_{x;t;k}$

Newly emerging unfunded liabilities are amortized by special payments spread over 15 years from each valuation date. Total contributions in this case are equal to the total normal cost plus special payments.

When there is a gain since the last valuation, we apply the gain to reduce the previously established special payments proportionally. In the case that there is a surplus after removing all previously established special payments, we allocate up to 5% of the net liability to a bu er required under the BC pension regulations and refer to the remaining surplus, if any, as "usable surplus". We determine two possible contribution reduction amounts by amortizing the usable surplus over a 15-year period and over a 25-year period. We establish the minimum contribution rate as the normal cost less the 15-year amortization of surplus. We also establish the maximum contribution rate as the normal cost less the 25-year amortization of surplus. We then apply the following algorithm.

1. If the contribution rate determined in the last valuation is lower than the minimum contribution rate determined in the current valuation, then the rate should be increased to be equal to the minimum contribution rate, resulting in 15-year amortization of the usable surplus.

2. If the contribution rate determined in the last valuation is greater than the maximum contribution rate determined in the current valuation, then the contribution rate is reduced to the maximum level, resulting in much slower (25-year) amortization of the usable surplus.

3.

cushions the valuation results against dramatic swings in market value. We are interested in the e ect that each of these stabilization techniques has on our results.

We refer to the stylized JSPP described in this chapter as JSPP1, which includes all emerging in the early years, generates additional surplus whenever the rate of return on the pension fund is positive. The absence of these "gains" under JSPP3 means slightly higher contributions in the long run. Since our stylized plan starts from a position without any surplus, a portion of the early gains under JSPP1 and JSPP2 goes to build up the rate stabilization reserve, which bene ts later cohorts. This represents a value transfer from early cohorts to later cohorts, which does not occur under JSPP3. Those who join JSPP3 early bene t from investment gains through the reductions on their contributions; those who join the plan 30 years from now are required to make more contributions, compared to members with same ages but under JSPP2.

Chapter 4

3. Retirement Dates:
   Normal retirement dates are the rst day of September following a member's 65th birthday. Early retirement is allowed on the rst day of any month after attained age 55.

4. Bene ts on Retirement:
   Members can apply their accumulated funds to the purchase of an annuity, or move their account balance to another registered plan.

4.2 Stylized DC Plan

In line with the SFU plan, our stylized DC plan sets up an individual account for each plan member, and applies the balance, consisting of the accumulated contributions and investment earnings, to purchase a guaranteed annuity at the member's retirement date. However, instead of contributing a xed percentage of salary, we assume contributions to the DC account are made at the same rates as to our stylized JSPP. That is, contribution rates can uctuate from year to year and scenario to scenario.

4.2.1 Contributions

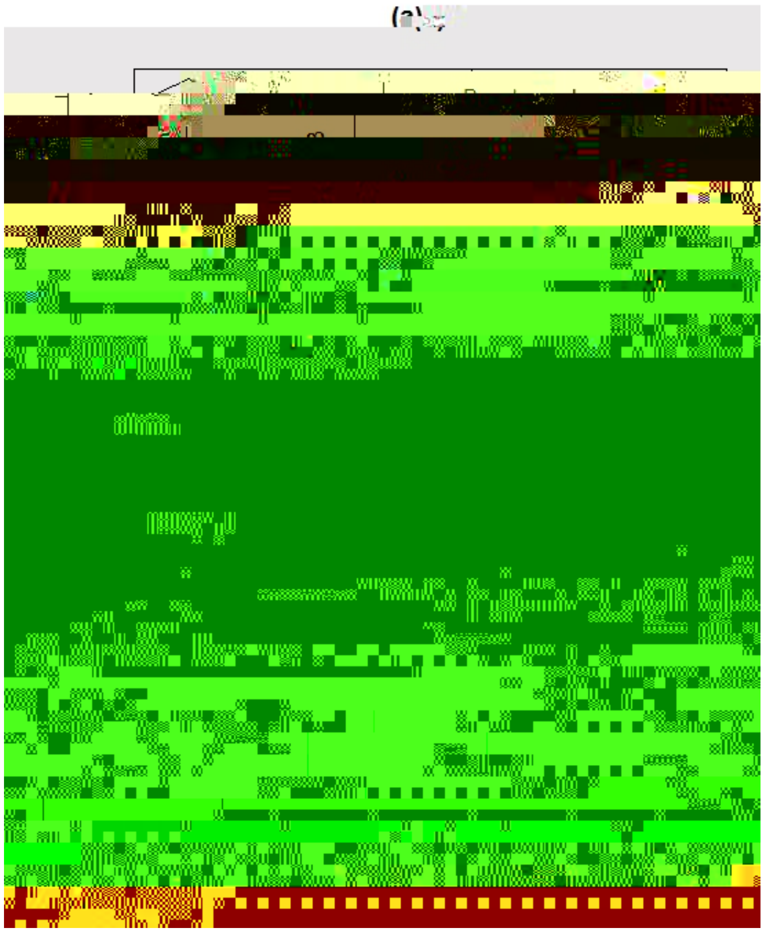
Annual contributions are made to the DC account as a percentage, c_t, of a member's annual salary, where c_t is as in the stylized JSPP. We also apply the same investment strategy as under the stylized JSPP with annual portfolio return r_t^P on $[t-1; t)$.

4.2.2 Pension Bene ts

The DC plan member uses the accumulated account value to purchase a guaranteed annuity from an insurance company upon his retirement. To project annuity purchase rates applicable in future years, we refer to the work of the CIA Committee on Pension Plan Financial Reporting (PPFRC). Every quarter, the PPFRC issues guidance regarding the "appropriate discount rate for estima36 Td [3(the)-2m7ttacfr4o7stimg.43R4o7stim0(purc)28(has6 Td [3(tad [3(tnon-inc
and the spreads recommended by the PPFRC (CIA, 2016). The "Annuity Conversion Rate" series in panel (b) is the sum of the two series from panel (a), representing the annuity purchase rate recommended by the PPFRC. The series labeled "10-year Bond" uses the same historical data as our VAR model. We see from panel (b) that the gap between the annuity purchase rate and the 10-year zero coupon yield is consistently around 1%.

Therefore, we estimate the annuity purchase rate applicable at time t

Figure 4.1: Relationship of annuity purchase rate and bond yields



Source: Statistics Canada. CANSIM series V39062. Table 176-0048: Bank of Canada, money market and other interest rates, daily; Canada; Government of Canada marketable bonds, over 10 years. http://www.bankofcanada.ca/rates/interest-rates/lookup-bond-yields/ . Accessed October 9, 2016.

Chapter 5

Performance Evaluation

5.1 Comparison Criteria

We investigate the performance of the two pension plans for identical twins. One twin joins the stylized JSPP, makes varying contributions to the fund, and collects guaranteed benefits after retirement; the other twin follows the same contribution pattern, manages the money in his own account, and transfers the accumulated savings to purchase an annuity certain with the same period as his sibling's pension benefit. Both twins are assumed to be 30 years old at entry with annual salary of $70,000.

We use two metrics to compare outcomes. The first uses the pension ratio, which is the ratio of DC pension to JSPP pension. Unlike in Blake, Cairns and Dowd (2001), the benefits under our JSPP1 and DC1 are directly comparable because they have the same contribution patterns. Our simulations generate an empirical distribution of possible pension ratios. The values of the pension ratios range from 0.40 at the lower end to 6.24 at the upper end. To make a comparison, we apply value-at-risk, which is widely used in studying tail risks. We specify one or more percentiles from our distribution, and compare these values with a target pension ratio of 1. The i^{th} percentile is the VaR at the $(100 - i)^{th}$ percent confidence level. If this percentile is greartain where Con_t is the consumption at time t. Prior to retirement, the twins have the same salary and the same contribution patterns, so their consumption is the same. Therefore, we only need to consider consumption during the retirement years, which is the annual pension payment B. The parameter γ represents the constant relative risk aversion level. We choose a relatively conservative $\gamma = 5$ which implies that "workers are ready to pay as much as 2.4% of their wealth to eliminate a fifty fifty risk to gain or loose 10% of their wealth" (Gollier, 2008).

The expected discounted utility of benefits is:

$$E_{r_e}(U) = E\left[\sum_{t=0}^{k-1} e^{-\delta t} u(\hat{B}_t)\right]; \tag{5.2}$$

where δ is the individual's time preference rate chosen as 0.04 following Cui et al. (2011), and $\hat{B}_t$ is the benefit received at time t, adjusted for the effect of inflation during the mem-

5.2 Numerical Results: Benchmark Comparison

We first look at the benchmark case (DC1 versus JSPP1). Value-at-risk statistics are in the first row of Table 5.2 panel (a). On average, the DC twin receives 1.59 times the retirement pension of his JSPP twin. If we want a reliable indicator of how risky the DC plan can be, we can look at the 5% quantile which is 0.8564. It indicates a 5% chance that the pension ratio will be less than 86%. However, as the required confidence level decreases, the DC plan becomes more attractive. For example, if we take the 75% confidence level, then the DC plan outperforms JSPP. The confidence level at which the DC twin's pension is the same as the JSPP twin's (i.e., a VaR of 1) is 88.07%. In conclusion, whether or not the DC plan is more competitive than the JSPP will depend on the choice of VaR confidence level.

The expected discounted utility summarized in Table (5.2) panel (b) supports that, with the same contribution pattern, the DC plan can be a better choice from a welfare perspective, based on the assumed risk tolerance of the twins. Note that the choice of time preference rate, δ, does not affect the ordering of the pension plans because the benefits are fixed after retirement, so the terms relating to δ can be factored out of the expected value:

$$
\begin{aligned}
E_{r_e}(U) &= E\left[\sum_{t=0}^{\infty} e^{-\delta t}\, u(\hat{B})\right] \\
&= E\left[u(\hat{B})\, \ddot{a}_{\overline{\infty}|}\right] \\
&= \ddot{a}_{\overline{\infty}|}\, E\left[u(\hat{B})\right].
\end{aligned}
\tag{5.6}
$$

As a result, changing δ changes the expected discounted utility of both options by the same proportion.

5.3 Numerical Results: Alternative Designs

To recap, we display the alternative JSPP and DC designs in Table 5.1. From DC1 to DC2, the annuity purchase capacity is improved with higher valuation rates. From JSPP1 to JSPP2, asset values experience more volatility. From JSPP2 to JSPP3, surplus in the valuation is amortized sooner and more completely.

Table 5.1: Comparison of alternative DC and JSPP designs

	DC1	DC2	JSPP1	JSPP2	JSPP3
Membership	Single Member				

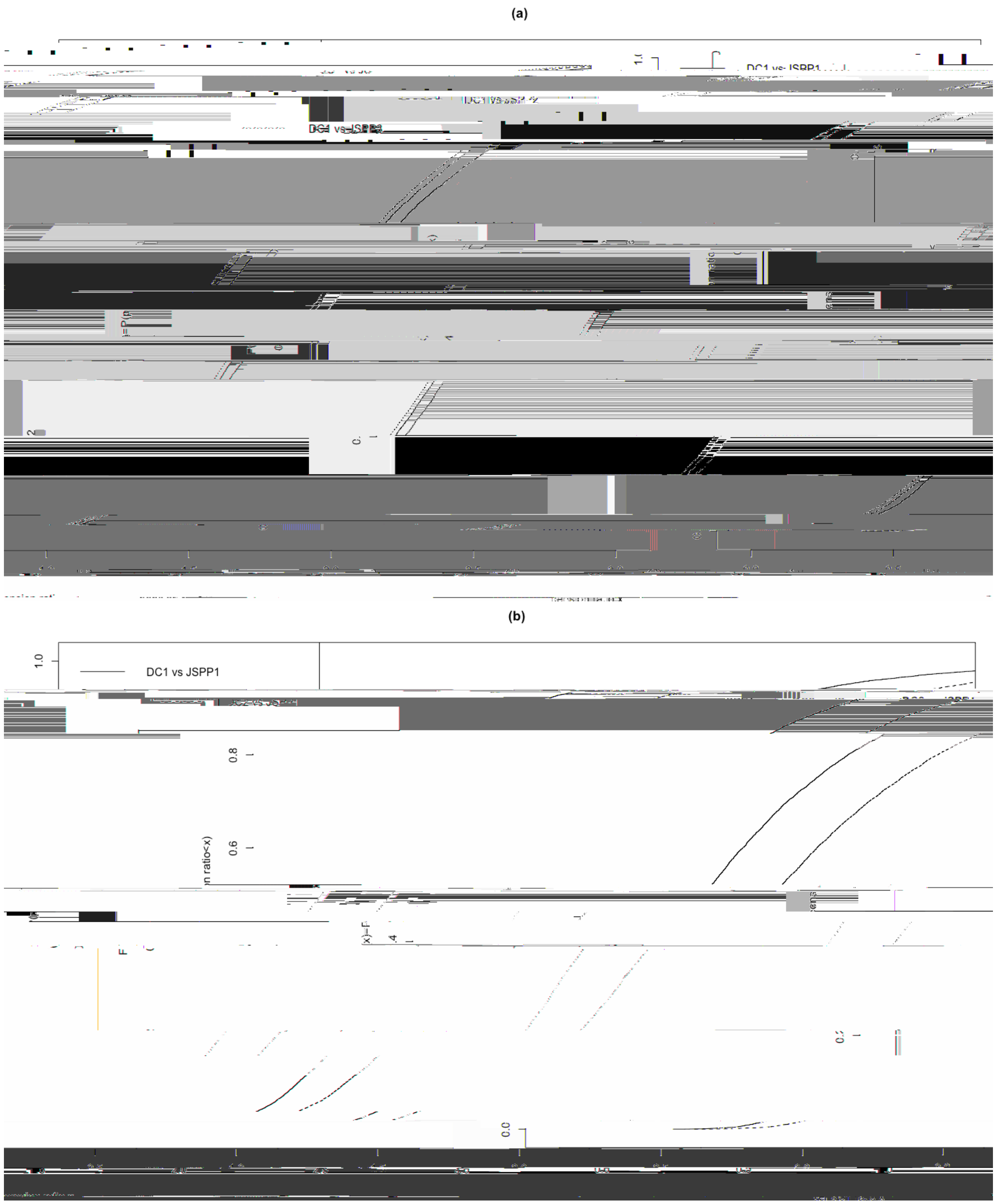
Figure 5.1 shows the empirical cumulative distribution functions of the pension ratio for different pairs of pension plan designs. A curve that lies further to the right means that the DC design is more likely to deliver higher pensions at retirement than the corresponding JSPP design. From Figure 5.1 (a), the DC plan following the contribution patterns of JSPP3 shows less advantages than the DC plan following the contribution patterns of JSPP1. In this sense, the rate stabilization structure has secured the stylized JSPP both on the asset and liability sides. From Figure 5.1 (b), available benefits increase if the DC twin is able to purchase his retirement annuity at a better price.

We can only compare expected discounted utility under the same contribution pattern. Results in Table 5.2 panel (b) prove that a cheaper annuity at retirement leads to welfare gains.

Table 5.2: Pension ratios and expected utility for different DC and JSPP designs

(a) value-at-risk statistics							
	Mean	SD	VaR				Critical Value 1 VaR
			50%	75%	90%	95%	
DC1 vs JSPP1	1.5857	0.5887	1.4762	1.1835	0.9637	0.8564	88.07%
DC1 vs JSPP2	1.5368	0.5709	1.4284	1.1485	0.9361	0.8260	86.23%
DC1 vs JSPP3	1.5124	0.5630	1.4104	1.1285	0.9181	0.8050	84.94%
DC2 vs JSPP1	1.8441	0.6669	1.7220	1.3880	1.1326	1.0070	95.28%
DC2 vs JSPP2	1.7873	0.6467	1.6670	1.3448	1.0999	0.9711	94.11%
DC2 vs JSPP3	1.7589	0.6380	1.6461	1.3240	1.0756	0.9480	93.25%
(b) expected discounted utility (salary scaled to 1/100,000)							
	$E_{r\ e}(U)$						
DC1	-18.00						
DC2	-9.43						
JSPP1	-35.22						

Figure 5.1: Cumulative distribution functions for di erent DC and JSPP designs (x_0)



5.4 Numerical Results: Alternative Economic Condition

To answer the question of whether today is the right time to switch from a DC plan to a JSPP, we investigate the case that the twins join the stylized plans when economic variables are above their long-term means. Speci cally, simulations of economic scenarios start from a state as far from the long term mean as we are today, but in the opposite direction. More precisely, we replace the time 0 state variable

$$x_0 = \quad (\quad x_0)$$

with

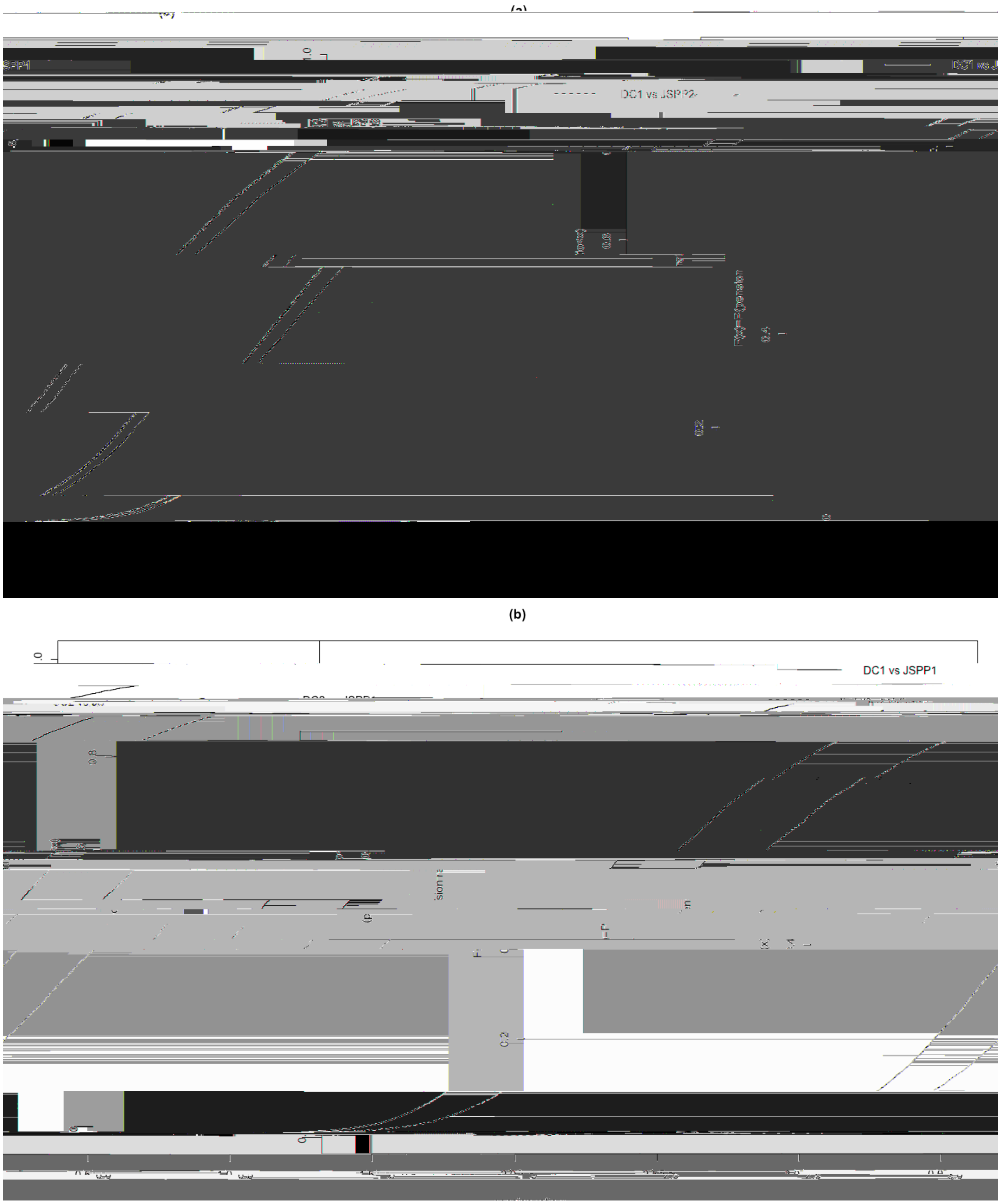
$$x_0^0 = \quad + (\quad x_0) = 2 \quad x_0 :$$

The resulting average normal costs and contribution rates are listed in Table 5.3 panels (a) and (b). Value-at-risk statistics and expected discounted utility can be found in panels (c) and (d). Figure 5.2 shows the empirical cumulative distributions of the pension ratios. Our observations are as follows.

Normal cost rates show a gradual upward shift in the rst 20-25 years, because interest improves the DC pension ratio since the cumulative distribution function curve moves to the right.

The expected discounted utility ranks the DC1, DC2 and JSPP1 designs in the same order as before. From a welfare perspective, the DC plan is more appealing, and a higher annuity purchase rate will transform the accumulated account value into a better lifetime bene t.

Figure 5.2: Cumulative distribution functions for di erent DC and JSPP designs (x_0^0)



the net value that each plan provides. Speci cally, the methodology used in Hoevenaars and Ponds (2008) and Lekniute at al. (2014) can be applied to estimate the value that market participants may pay for the net bene t stream, taking into account risks. This requires the estimation of the market price of risk, which can be achieved by extending the VAR model with an a ne term structure model of interest rates (Cochrane and Piazzesi, 2005). The resulting model, which we constrain to being arbitrage free, produces a stochastic discount factor (or pricing kernel), which can be used to discount both future contributions and bene ts. In the rest of this chapter we outline how this could be implemented based on Cochrane and Piazzesi (2005) and Hoevenaars (2008).

As suggested in Cochrane and Piazzesi (2005), the market prices of risk can be generated in an a ne model. In line with their VAR model which contains an intercept term, we rewrite equation (2.1) as:

$$x_{t+1} \quad = (\ \ x_t \quad) + P \ \ _{t+1}; \qquad (6.1)$$

and therefore,

$$x_{t+1} = (I \quad) \quad + \quad x_t + P \ \ _{t+1}; \qquad (6.2)$$

and we use $= (I \quad)$. The state variables included in x_t would need to be slightly di erent than described in Chapter 2. Stock return variables should be returns in excess of the short term interest rate and represent only the price appreciation. The corresponding dividend yields, considered as non-tradable assets on which the risk premium is zero, should appear as separate state variables rather than being part of the total equity return.

The pricing kernel has the following form:

$$M_{t+1} = \exp(\quad _0 \quad _1 x_t \quad \frac{1}{2} \ _t^T P P^T \ _t \quad _t^T P \ \ _{t+1}); \qquad (6.3)$$

where M_{t+1} is the one-period stochastic discount factor, and $_0 + \ _1 x_t$ is the short rate which is a ne in the state variables of the VAR. The innovation term $_{t+1}$ is the same as in equation (6.2). To keep consistency between the VAR model and the pricing kernel, we let $y_t^1 = \ _0 + \ _1 x_t$. The rst part of the stochastic discount factor, $\exp(\quad _0 \quad _1 x_t)$, represents the risk-free discount factor. The other component, $\exp(\quad \frac{1}{2} \ _t^T P P^T \ _t \quad _t^T P \ \ _{t+1})$, relates shocks in the state variables to the pricing kernel. We use $_t$ to represent the market price of risk and assume it has the following form:

$$_t = \ _0 + \ _1 x_t : \qquad (6.4)$$

The vector $_0$ accounts for the constant part of the risk premium, and the matrix $_1$ accounts for time-variation. Since the market price of risk is the excess expected return per unit of covariance, $_t$ is strictly positive. If the state of the economy is such that the market price of risk is high, the stochastic discount factor in (6.3) assumes a low value, all other things being equal.

The following section is in line with the description in Hoevenaars (2008). Asset pricing theory states that the price of an asset (P_t) is its expected discounted payo :

$$P_t = E_t(M_{t+1} X_{t+1}); \qquad (6.5)$$

where X_{t+1} is the asset payo . The price $P_t^{(n)}$ of an n-period nominal bond at time t has the form of:

$$P_t^{(n)} = E_t(M_{t+1} P_{t+1}^{(n \;\; 1)}): \qquad (6.6)$$

At the same time, the bond price can be expressed as an exponential a ne function of the state variables in the VAR model. More precisely, bond prices are given by

$$P_t^{(n)} = \exp(A_n + B_n^T x_t); \qquad (6.7)$$

and therefore, log bond prices $p_t^{(n)}$ becomes a linear function of the state variables:

$$p_t^{(n)} = A_n + B_n^T x_t: \qquad (6.8)$$

The scalar A_n and the vector B_n follow the di erence equations:

$$\begin{aligned} A_{n+1} &= A_n + B_n^T(\quad P P^T \;\; _0) + \frac{1}{2} B_n^T P P^T B_n \quad\;\; _0 \\ B_{n+1}^T &= B_n^T(\quad P P^T \;\; _1) \quad\;\; _1 \end{aligned} \qquad (6.9)$$

with $A_0 = B_0 = 0$ as $p_t^{(0)} = 0$. These di erence equations can be derived by induction using equation (6.8); see Appendix D for details. The equations above show that the constant part of the risk premium $\;\; _0$ in uences A_n, and the time-varying component $\;\; _1$ in uences B_n.

Once $_0$ and $_1$ are estimated, we can simulate the state variable x_t as well as the stochastic discount factor M_t. Bond returns are calculated by equation (2.14), but the projected log price $p_t^{(n)}$ is based on equation (6.10). The new performance criterion is the expected stochastic present value of net benefits ($ESPV$), which is the market value at time 0 of benefits to be received from the plan less contributions to be paid to the plan, adjusted for risk. More precisely, $ESPV$ is the average of the stochastic present values determined under each scenario:

$$SPV = CF_0 + \sum_{t=1}^{r+l_e-1} \left[\prod_{h=12(t-1)+1}^{12t} M_h \right] CF_t \tag{6.11}$$

where M_h corresponds to the one-month stochastic discount factor applicable to month h with $h = 1$ corresponding to January 2017 and t is measured in years. A positive CF_t means a cash inflow (the benefit payment at time t), and a negative CF_t means a cash outflow (contribution made at time t). Under our stylized JSPP and stylized DC plan, there are only cash outflows before retirement and only cash inflows afterwards. A positive $ESPV$ means that an individual pays less than the market value of the benefit stream he could receive, and a negative $ESPV$ means that the pension plan requires an individual to pay

Chapter 7

Conclusion

In this study, we compare the performance of two stylized pension plans: a DC plan based on the Simon Fraser University pension plan for faculty members and a jointly-sponsored pension plan replicating some features of the B.C. College Pension Plan. A VAR(1) model is used to generate economic scenarios. We investigate pension ratios and expected discounted utilities for a representative cohort. We also conduct analysis on alternative plan designs and di erent economic conditions to understand the two plans more comprehensively. We nd that the DC outcomes are volatile, but risk is mostly on the upside when contribution rates mimic those under the JSPP. In terms of expected discounted utilities, the DC plan wins even for a conservative member. In addition, the impact of smoothing mechanisms under the JSPP is quite small. Finally, the DC plan would do even better if insurance companies were subject to the same rates as pension plans and if simulations were started in "opposite" economic conditions.

In designing our stylized DC plan and our stylized JSPP, we make some simplifying assumptions. For example, expenses are completely ignored. In reality, the total cost of investment management and pension administration for large public funds tends to be much less than what DC plan members commonly pay. Another conservative assumption on the stylized JSPP is that investment returns are based on an index, while actual public sector pension plans o er more e cient and less volatile asset allocation strategies. In addition, we focus on the guaranteed basic pension bene t while conditional in ation protection is an important characteristic of JSPPs in general, and the College Pension Plan in particular.

Potential future work includes more sophisticated asset models, simulation of the In ation Adjustment Account, as well as the implementation of the market-value based performance criterion described in Chapter 6.

RRSP-PENSION-FUNDS-15/Broadbent%20doc.2000-%20Palumbo%20Woodman%202006%20Shift%20of%20DC%20to%20DB%20plans.pdf

[9] Brown, R.L., & McInnes, C. (2014). Shifting Public Sector DB Plans to DC-The experience so far and implications for Canada.Canadian Public Pension Leadership Council, October, 9. Retrieved from https://www.optrust.com/documents/DB-vs-DC-plans-research-paper.pdf .

[10] Canadian Institute of Actuaries. (2014). Pension Experience Subcommittee.Final Report of Canadian Pensioners' Mortality. Retrieved from http://www.cia-ica.ca/docs/default-source/2014/214013e.pdf .

[11] Canadian Institute of Actuaries. (2015). Task Force on Target Bene t Plans. Report of the Task Force on Target Bene t Plans. Retrieved from http://www.cia-ica.ca/docs/default-source/2015/215043e.pdf .

[12] Canadian Institute of Actuaries. (2016). Committee on Pension Plan Financial Reporting. Educational Note Supplement: Guidance for Assumptions for Hypothetical Wind-Up and Solvency Valuations Update Retrieved from http://www.cia-ica.ca/docs/default-source/2016/216111e.pdf .

ed from
[13] Cochrane, J.H., & Piazzesi, M. (2005). Bond Risk Premichrane, .

[19] Gollier, C. (2008). Intergenerational Risk-sharing and Risk-taking of A Pension Fund. Journal of Public Economics, 92(5), 1463-1485. doi: 10.1016/j.jpubeco.2007.07.008.

[20] Heidelberg, L. (2005). New Introduction to Multiple Time Series Analysis. Berlin: Springer Science & Business Media.

[21] Hoevenaars, R.P. (2008). Strategic asset allocation & asset liability management. PhD Dissertation. Universiteit Maastricht. Retrieved from http://digitalarchive.maastrichtuniversity.nl/fedora/get/guid:8e6f53dc-9a2d-4200-b103-4fd65302f516/ASSET1.

[22] Hoevenaars, R.P., & Ponds, E.H. (2008). Valuation of intergenerational transfers in funded collective pension schemes. Insurance: Mathematics and Economics, 42(2), 578-593. doi: 10.1016/j.insmatheco.2007.06.007.

[23] Kristjanson, F., & Darrach, A. (2012). Challenges for Jointly Sponsored Pension Plan. 10th Annual OBA Pensions and Bene ts Hot Spots, Toronto. Retrieved from http://www.cavalluzzo.com/docs/default-source/publications/2012-05-01-challenges-for-jointly-sponsored-pension-plans-(freya-kristjanson-and-amanda-darrach)-pensions-law.pdf?sfvrsn=2 .

[24] Lekniute, Z., Beetsma, R.M., & Ponds, E.H. (2014). A Value-based Approach to the Redesign of US State Pension Plans. doi: 10.2139/ssrn.2438637.

[25] McCarthy, D. (2003). A life-cycle analysis of de ned bene t pension plans. Journal of Pension Economics and Finance, 2(02), 99-126. doi: 10.1017/S1474747203001288.

[26] Municipal Pension Plan. (n.d.). An in-depth look at BC public sector pension plans. Source: www.pensionbc.ca retrieved Feb 27, 2017.

[27] PBI Actuarial Consultants Ltd. (2015). Comparison of De ned Bene t and De- ned Contribution Pension Plans. Retrieved from http://www.sfufa.ca/wp-content/uploads/2014/07/actuarial-report-march-15.pdf .

[28]

[30] SFU. (2013). SFU Pension Plan for Academic Sta Summary. Retrieved from https://www.sfu.ca/content/dam/sfu/human-resources/forms-documents/pensions/academic_pension_annual_reports/Academic%20Summary%20Brochure%20FINAL%202013.pdf

[31] SFU. (2015). SFU 2015 Annual Report Academic Pension Plan. Retrieved from https://www.sfu.ca/content/dam/sfu/human-resources/forms-documents/pensions/academic_pension_annual_reports/2015%20Academic%20Plan%20Annual%20Report.pdf

[32] Statistics Canada. (n.d.). Table 326-0020 Consumer Price Index, mo1of

Appendix A

Membership of Stylized JSPP

The following three tables display membership information that we use to project the styl-

Table A.2: Active member data for the stylized JSPP

	Male			Female		
Age	Number	Average Salary ($)	Service	Number	Average Salary ($)	Service
24	7	63; 059	0.1	11	36; 296	0.5
27	50	59; 443	0.7	113	60; 167	0.9
30	109	68; 065	1.5	181	66; 446	1.9
33	109	68; 065	1.5	181	66; 446	1.9
36	205	72; 709	29	317	72; 166	3.2
39	205	72; 709	29	317	72; 166	3.2
42	600	78; 185	4.4	843	76; 446	4.6
45	366	81; 332	6.3	497	78; 852	6.5
48	366	81; 332	6.3	498	78; 852	6.5
51	465	84; 675	9.5	533	81; 122	8.4
54	464	84; 675	9.5	533	81; 122	8.4
57	886	86; 612	11	1038	83; 193	10.8
60	1069	87; 697	123	985	82; 709	11.9

Table A.3: Pensioners data for the stylized JSPP

	Annual Pensions ($)		
Age	Male	Female	
60	2,473,000	3,386,000	
62	12,161,000	15,030,000	
67	21,305,000	23,214,000	
72	21,629,000	13,405,000	23,214,000

When t

Appendix C

Amortization Method

As discussed in section (3.3.5), any unfunded liability is amortized over the next 5 valuations (15 years). In the case of a surplus, only the surplus in excess of 5% of the net liability (the "usable surplus") is amortized. We determine new minimum and maximum contribution levels at each valuation. An increase of contribution rate is required if the contribution rate established in the previous valuation is smaller than the new minimum contribution rate. A reduction of contribution rate is approved if the contribution rate established in the previous valuation is greater than the maximum contribution rate. The contribution rate is unchanged otherwise.

Notations that are used in this section:

- AV_h : actuarial value of assets established in valuation h (at time $3h$);
- AL_h^{EAN} : actuarial liability established in valuation h;
- $TSal_h$: total annual salaries in valuation h; equal to $\sum_{x}\sum_{k6h} Sal_{x;3h;k}$;
- $Surp_h$: surplus (unfunded liability) from pure assets in valuation h;
- $SP_{h;m}$: annual special payment rst established in valuation $h \quad m$ to amortize an unfunded liability, and still applicable in valuation h; $(m = 0; 1; 2; 3; 4)$;
- $Surp_h$: adjusted surplus, with present value of special payments being included as an additional asset,
- spr_h : special payment rate from $Surp_h$ in valuation h;
- spr_h : special payment rate from $Surp_h$ in valuation h;
- c_h^{min} : the contribution rate arising after amortizing any usable surplus surplus over a 15-year period,
- c_h^{max} : the contribution rate arising after amortizing any usable surplus surplus over a 25-year period,
- c_h : contribution rate before adding the non-negative constraint in valuation h;
- c_h : nal contribution rate established in valuation h:

When $h = 0$,

$$Surp_0 = Surp_0 = 0; \tag{C.1}$$

$$SP_{0;0} = SP_{0;1} = SP_{0;2} = SP_{0;3} = SP_{0;4}; \tag{C.2}$$

$$c_h = c_0^{NC}: \tag{C.3}$$

When $h = 1; 2; :::$

$$Surp_h = AV_h \quad AL_h^{EAN}: \tag{C.4}$$

If $Surp_h > 0$, the actuarial value of plan assets is enough to nance future liabilities without the need for any special payments in excess of the normal cost. We remove any previously established special payments:

Wh

$$SP_{h;m} = 0; m = 0; 1; 2; 3; 4; \tag{C.5}$$

$$spr_h = 0: \tag{C.6}$$

NC 6AL

We establish the minimum and maximum contribution rates as the normal cost rate less the amortization of the usable surplus over a 15-year period and a 25-year period, 25-year period,pSP let the new special payment be

$$SP_{h;0} = \frac{Surp_h}{\ddot{a}_{\overline{15}|y_h^V}}; \qquad (C.14)$$

and continue the special payments established in prior valuations as original scheduled:

$$SP_{h;m} = SP_{h-1;m-1}; \quad m = 1;2;3;4: \qquad (C.15)$$

We convert these special payment amounts to rates of pay:

$$spr_h = \frac{Surp_h}{\ddot{a}_{\overline{15}|y_h^V} \; TSal_h}; \qquad (C.16)$$

$$spr_h = spr_{h-1} + spr_h: \qquad (C.17)$$

The new contribution rate is then the normal cost rate plus the total special payment rate including the newly established portion:

$$c_h = \max(c_h^{NC} + spr_h; 0): \qquad (C.18)$$

If $Surp_h > 0$, the actuarial value of plan assets plus future special payments are

Appendix D

Recursive Bond Price

As in Ang and Piazzesi (2003), to derive the equations in (6.9), we rst note that for a one-period bond:

$$\begin{aligned} P_t^{(1)} &= E_t(M_{t+1}) \\ &= E_t[\exp(\quad {}_0 \quad {}_1 x_t \quad ^{1} \end{aligned}$$

Again, the normally distributed $_{t+1}$ implies that

$$
\begin{aligned}
E_t\left[\exp\{(B_n^T \quad {}_t^T)P \quad {}_{t+1}\}\right] &= \exp[\frac{1}{2}(B_n^T \quad {}_t^T)PP^T(B_n^T \quad {}_t^T)^T] \\
&= \exp[\frac{1}{2}B_n^T PP^T B_n \quad B_n^T PP^T \quad {}_t + \frac{1}{2} \, {}_t^T PP^T \quad {}_t]
\end{aligned}
\qquad (D.5)
$$

Taking (D.5) into (D.4), we get

$$
\begin{aligned}
P_t^{(n+1)} &= \exp(\quad {}_0 + A_n + B_n^T \quad + (B_n^T \quad {}_1)x_t + \frac{1}{2}B_n^T PP^T B_n \quad B_n^T PP^T \quad {}_t) \\
&= \exp(\quad {}_0 + A_n + B_n^T \quad + (B_n^T \quad {}_1)x_t + \frac{1}{2}B_n^T PP^T B_n \quad B_n^T PP^T(\quad {}_0 + \quad {}_1 x_t)) \\
&= \exp(\quad {}_0 + A_n + B_n^T(\quad PP^T \quad {}_0) + \frac{1}{2}B_n^T PP^T B_n + (B_n^T \quad {}_1 \quad PP\text{-2091 Tf 137 T101 Tf 17.0}
\end{aligned}
$$