

Thermodynamic Metrics and Optimal Paths

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Abstract: We study the thermodynamic metrics and optimal paths in the space of parameters for a system of particles. The metrics are derived from the Fisher information matrix and the optimal paths are found by minimizing the action. The results are compared with the known results for the Ising model and the XY model.

Introduction: The study of thermodynamic metrics and optimal paths in the space of parameters is a topic of great interest in the field of statistical mechanics. In this paper, we consider a system of particles and study the thermodynamic metrics and optimal paths in the space of parameters. The metrics are derived from the Fisher information matrix and the optimal paths are found by minimizing the action. The results are compared with the known results for the Ising model and the XY model.

1.1. Thermodynamic Metrics: The thermodynamic metrics are derived from the Fisher information matrix. The Fisher information matrix is defined as $F_{ij} = \langle \partial_i \ln Z \partial_j \ln Z \rangle$, where Z is the partition function and ∂_i is the derivative with respect to the parameter i . The thermodynamic metrics are then defined as $G_{ij} = F_{ij}^{-1}$.

1.2. Optimal Paths: The optimal paths are found by minimizing the action. The action is defined as $S = \int \dot{x}^i G_{ij} \dot{x}^j dt$, where x^i are the parameters and \dot{x}^i are their time derivatives. The optimal paths are found by solving the Euler-Lagrange equations.

1.3. Results: We study the thermodynamic metrics and optimal paths for a system of particles. The results are compared with the known results for the Ising model and the XY model. We find that the thermodynamic metrics and optimal paths for the system of particles are similar to those for the Ising model and the XY model.

1.4. Conclusion: In conclusion, we have studied the thermodynamic metrics and optimal paths in the space of parameters for a system of particles. The results are compared with the known results for the Ising model and the XY model. We find that the thermodynamic metrics and optimal paths for the system of particles are similar to those for the Ising model and the XY model.

$$p(\mathbf{x}; \boldsymbol{\lambda}) = \frac{1}{Z(\boldsymbol{\lambda})} \exp \left[-\beta E(\mathbf{x}; \boldsymbol{\lambda}) \right]$$

$$Z(\boldsymbol{\lambda}) = \sum_{\mathbf{x}} \exp \left[-\beta E(\mathbf{x}; \boldsymbol{\lambda}) \right]$$

$$\ln Z(\boldsymbol{\lambda}) = \ln \sum_{\mathbf{x}} \exp \left[-\beta E(\mathbf{x}; \boldsymbol{\lambda}) \right]$$
$$= \ln \sum_{\mathbf{x}} \exp \left[-\beta \sum_i \left(-J_{ij} x_i x_j - \sum_a h_a x_a \right) \right]$$
$$= \ln \sum_{\mathbf{x}} \exp \left[\beta \sum_i \left(\sum_j J_{ij} x_i x_j + h_i x_i \right) \right]$$
$$= \ln \sum_{\mathbf{x}} \exp \left[\beta \sum_i x_i \left(\sum_j J_{ij} x_j + h_i \right) \right]$$
$$= \ln \sum_{\mathbf{x}} \exp \left[\beta \sum_i x_i \tilde{h}_i \right]$$
$$= \ln \sum_{\mathbf{x}} \prod_i \exp \left[\beta x_i \tilde{h}_i \right]$$
$$= \ln \prod_i \sum_{x_i = \pm 1} \exp \left[\beta x_i \tilde{h}_i \right]$$
$$= \sum_i \ln \left(2 \cosh \beta \tilde{h}_i \right)$$
$$= \sum_i \ln \left(2 \cosh \beta \left(\sum_j J_{ij} x_j + h_i \right) \right)$$
$$= \sum_i \ln \left(2 \cosh \beta \left(\sum_j J_{ij} \langle x_j \rangle + h_i \right) \right)$$
$$= \sum_i \ln \left(2 \cosh \beta \left(\sum_j J_{ij} \langle x_j \rangle + h_i \right) \right)$$

$$\mathcal{P}_{\text{ex}}(t_0) = \int_{-\infty}^{t_0} dt' \frac{d\boldsymbol{\Sigma}^{(\boldsymbol{\lambda}(t_0))}(t_0 - t')}{dt'} \cdot [\boldsymbol{\lambda}(t_0) - \boldsymbol{\lambda}(t')]$$

$$\mathcal{P}_{\text{ex}}(t_0) = \int_{-\infty}^{t_0} dt' \boldsymbol{\Sigma}^{(\boldsymbol{\lambda}(t_0))}(t_0 - t') \cdot \frac{d\boldsymbol{\lambda}}{dt'} ;$$

$$\int_{t' = -\infty}^{t' = t_0} \dots$$

