SIMON FRASER UNIVERSITY

MATH 157 D100 Midterm II (A) Solution

Wednesday, November 9, 2016, 11:30-12:15 (only 45 minutes)

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Last Name____________________

Given Name(s)____________________

Student # ____________________

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INSTRUCTIONS

1. Write your last name, given name(s), student number, and SFU email ID in the box above.
2. Sign your name in the box provided.
3. This exam has 5 questions on 6 pages. Please check to make sure your exam is complete.
4. This is a closed book exam.
5. Only approved calculators are permitted.
6. Ask for clari cation if you cannot understand a question or there appears to be an error.
7. Use the reverse side of the previous page if you need more room for your answer.
8. You may lose marks if your explanations are incomplete or poorly presented.
9. Please write rmly so that scanning picks up on your writing. Cross out unwanted work (do not erase) and continue on the previous blank page if you need more space.

Question	Maximum	Score
1	6	
2	9	
3	4	
4		

[3] 1. (a) Let $f(x) = 2x^4 - 4x^3 + 7$.

(i) Find $f'(x)$.

Solution: $f'(x) = 8x^3 - 12x^2$.

(ii) Find the iterative formula for $f(x)$ in the Newton-Raphson method. Do not simplify your formula.

Solution:

$$x_{n+1} = x_n - \frac{2x_n^4 - 4x_n^3 + 7}{8x_n^3 - 12x_n^2}$$

[3] (b) Suppose the iterative formula for $f(x)$ in the Newton-Raphson method is

$$x_{n+1} = \frac{4x_n^5 + 1}{5x_n^4 + 1}, \quad n = 0, 1, 2, \ldots$$

Let $x_0 = 0$. Find x_1, x_2 and x_3.

Solution:

$$x_1 = \frac{4(0)^5 + 1}{5(0)^4 + 1} = 1.$$

$$x_2 = \frac{4(1)^5 + 1}{5(1)^4 + 1} = \frac{5}{6}.$$

$$x_3 = \frac{4(5/6)^5 + 1}{5(5/6)^4 + 1} = \frac{10138}{13263} = 0.7643821157$$

[4] 3. Given the average cost function is

$$\overline{C}(x) = 200 + \frac{300{,}000}{x},$$

for some product $x > 0$ and the demand for the product is

$$p = -0.04x + 800, \quad 0 \le x \le 20{,}000$$

(i) Find the total cost function $C(x)$.

Solution: $C(x) = x\overline{C}(x) = 200x + 300{,}000$.

(ii) Find the total revenue function $R(x)$.

Solution: $R(x) = px = 800x - 0.04x^2$.

(iii) Find the profit function $P(x)$.

Solution: $P(x) = R(x) - C(x) = 800x - 0.04x^2 - (200x + 300{,}000) = 600x - 0.04x^2 - 300{,}000$.

(iv) Find the marginal profit function $P'(x)$.

Solution: $P'(x) = 600 - 0.08x$.

[3] 4. Use differentials to find an approximation of the value $\sqrt[3]{26.99}$.

Solution: Let $y = f(x) = x^{1/3}$. Then $f'(x) = \frac{1}{3}x^{-2/3}$. The differential is

$$dy = \frac{1}{3}x^{-2/3}dx.$$

Let $x_1 = 27$ and $x_2 = 26.99$. Then $\Delta x = dx = 26.99 - 27 = -0.01$.
Since $f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$, so

$$\begin{aligned}
\Delta y &= f(26.99) - f(27) \\
&= \sqrt[3]{26.99} - \sqrt[3]{27} \\
&= \sqrt[3]{26.99} - 3 \\
&\approx dy \\
&= \frac{1}{27}(-0.01).
\end{aligned}$$

Hence $\sqrt[3]{26.99} \approx 3 - \frac{1}{2700} = \frac{8099}{2700} = 2.99962963$

[2] 5. (a) By implicit differentiation, find $\frac{dy}{dx}$ for the relation given by

$$x^2 y^{\frac{1}{2}} = x + 2y^3:$$

Solution:

$$2xy^{\frac{1}{2}} + x^2 \frac{1}{2} y^{\ \frac{1}{2}} y^0 = 1 + 6y^2 y^0$$

$$6y^2 \quad \frac{x^2}{2y^{\frac{1}{2}}} \quad y^0 = 2xy^{\frac{1}{2}} \quad 1$$

$$y^0 = \frac{2xy^{\frac{1}{2}} \quad 1}{6y^2 \quad \frac{x^2}{2y^{\frac{1}{2}}}}:$$

[6] (b) Find the first derivative for the following functions. Do not simplify your answer.

(i) $s = \dfrac{t^2 + 3}{\cos(t) + 1}$.

Solution: $\dfrac{ds}{dt} = \dfrac{(\cos(t) + 1)(2t) \quad (t^2 + 3)(\quad \sin(t))}{(\cos(t) + 1)^2}$.

(ii) $y = \ln(\sin^2(x^2 + 1))$.

Solution: $\dfrac{dy}{dx} = \dfrac{1}{\sin^2(x^2 + 1)} \quad 2\sin(x^2 + 1)(\cos(x^2 + 1))(2x)$.