BLUE PAPERS

SIMON FRASER UNIVERSITY

[7] **1.** Find $\int \frac{x+11}{x^2+4x-5}\,dx.$

$x^2+4x-5=0$

2. For each of the following two improper integrals, determine whether it is convergent or divergent. If the integral is convergent, determine its value.

[4] (a) $\int_1^4 \frac{2}{(x-3)^2}\,dx$

$$\int_1^4 \frac{2}{(x-3)^2}\,dx = \int_1^3 \frac{2}{(x-3)^2}\,dx + \int_3^4 \frac{2}{(x-3)^2}\,dx =$$

$$= \lim_{c\to 3^-} \int_1^c \frac{2}{(x-3)^2}\,dx + \lim_{c\to 3^+} \int_c^4 \frac{2}{(x-3)^2}\,dx =$$

$u = x-3$
$du = dx$

$$= -\lim \left[-\frac{2}{x-3}\right]^c + \lim \left[-\frac{2}{x-3}\right]^4$$

The integral is divergent.

[4] (b) $\int_5^\infty \frac{2}{(x-3)^2}\,dx$

$$\int_5^\infty \frac{2}{(x-3)^2}\,dx = \lim_{c\to\infty} \int_5^c \frac{2}{(x-3)^2}\,dx =$$

[7] **3.** Evaluate $\int x^2 \cos x \, dx$.

$u = x^2, \quad v' = \cos x, \quad v = \sin x$

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx =$$

$u = 2x, \quad v' = \sin x, \quad v = -\cos x$

$$= x^2 \sin x - \left(2x(-\cos x) - \int 2(-\cos x)\,dx\right) =$$

$$= x^2 \sin x + 2x \cos x - \int 2 \cos x \, dx =$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

[4] 4. Use the trapezoidal rule with $n = 4$ intervals to approximate the value

[7] 5. Solve the differential equation

$$\frac{dy}{dx} = 2\sqrt{y}$$

with the initial condition $y(0) = 4$.

$$\frac{dy}{2\sqrt{y}} = dx$$

$$\int \frac{dy}{2\sqrt{y}} = \int dx$$

$$\sqrt{y} = x + C$$

$$y(0) = 4 \Rightarrow \sqrt{4} = 0 + C \Rightarrow C = 2$$

$$\sqrt{y} = x + 2$$