BLUE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Midterm 1

3 February 2010, 08:30–09:20

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1. Indicate whether the following statements are true (T) or false (F). Justifications are <u>**not required.**</u>

Assume that $f(x)$ is continuous in the intervals of integration.

[redacted]

A statement containing general constants a, b, c and functions f, g is true if and only if it holds for *all admissible choices* that you can make for a, b, c, f, g.

[1] (a) T If $f(x) \geq c$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq c(b-a)$.

[1] (b) T $\int_a^b (f(x) - g(x))\,dx = \int_a^b f(x)\,dx + \int_b^a g(x)\,dx$

[1] (c) F $\int_a^b (f(x))^2\,dx = \left(\int_a^b f(x)\,dx\right)^2$ consider $f(x)=1$

[1] (d) F $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$ +

29

[4] **2.** Determine the average value of $f(x) = 4x^3 + 5$ in the interval $[1, 4]$.

$$f_{avg} = \frac{1}{4-1}\int_1^4 4x^3 + 5 \, dx =$$

$$= \frac{1}{3}\left[x^4 + 5x\right]_1^4 = \frac{1}{3}(276 - 6) = 90$$

[5] **3.** Graph the finite region that is enclosed by the curves $y = \sqrt{x}$, $y = -\frac{x}{2}$

[4] 4. (a) Express the sphere of radius r as a solid of revolution. (This involves

specifying a certain function and an interval on the x-axis.)

[8] 5. Find $\int \frac{(\sin x)^3}{\cos x}\,dx$.

Hint: Split the numerator into two factors, and use $(\sin x)^2+(\cos x)^2=1$

$$\int \frac{(\sin x)^3}{\cos x}\,dx = \int \frac{(\sin x)^2\cdot \sin x}{\cos x}\,dx =$$

[redacted]

$$= \int \frac{\left(1-(\cos x)^2\right)\cdot \sin x}{\cos x}\,dx =$$

$u = \cos x$

$$\int (1-u^2)(-du)$$

[8] **6.** Evaluate $\int_3^4 \frac{5}{x\sqrt{\ln(3x)}}\,dx$.

$$u = \ln(3x)$$

$$\frac{du}{dx} = \frac{1}{3x}\cdot 3$$

$$du = \frac{dx}{x}$$

$$\int_3^4 \frac{5}{x\sqrt{\ln(3x)}}\,dx = \int_{\ln 9}^{\ln 12} \frac{5\,du}{\sqrt{u}} =$$

$$= 5\int_{\ln 9}^{\ln 12} u^{-1/2}\,du = 5\left[\frac{u^{1/2}}{\frac{1}{2}}\right]_{\ln 9}^{\ln 12} =$$