BLUE PAPERS

SIMON FRASER UNIVERSITY

MATH 155 Final Exam

23 April 2010, 15:30–18:30

Last Name ______________________

Given Name(s) ______________________

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INSTRUCTIONS

1. **Do not open this booklet until told to do so.**
2. Fill out the box in the upper right corner of this page.
3. This exam has 11 questions on 11 pages. Check to make sure that your exam is complete.
4. No book, paper or device other than usual writing in- [illegible]

Question	Maximum	Score
1	8	
2	8	

[8] **1.** Evaluate $\int_0^1 xe^{-x^2}\,dx$.

$$\int^1 xe^{-x^2}\,dx =$$

$u = -x^2$

$\frac{du}{} = -2x$

[8] **2.** Evaluate $\int_{-2}^{0} xe^{-x}\,dx$.

$$\boxed{u = x, \quad v' = e^{-x}, \quad v = -e^{-x}}$$

$$\int_{-2}^{0} xe^{-x}\,dx = \left[x(-e^{-x})\right]_{-2}^{0} - \int_{-2}^{0} -e^{-x}\,dx =$$

$$= 0 - (-2)(-e^{2}) + \int_{-2}^{0} e^{-x}\,dx =$$

$$= -2e^{2} + \left[-e^{-x}\right]_{-2}^{0} = -2e^{2} - 1 - (-e^{2}) =$$

$$= -e^{2} - 1 \approx -8.389$$

[10] **3.** Evaluate $\displaystyle\int \frac{8x^2+4x-6}{4x^2+1}\,dx$.

$$\frac{8x^2+4x-6}{4x^2+1} = 2 + \frac{4x-8}{4x^2+1} \quad \text{(long division)}$$

$$\int \frac{8x^2+4x-6}{4x^2+1}\,dx = \int 2\,dx + \int \frac{4x-8}{4x^2+1}\,dx =$$

[0] 4. Compute the volume of the intersection of two spheres of radius 6 cm that

are positioned such that the distance of their centres is 6 cm. (Hint: Place

5. Suppose that at time t an object has temperature $T(t)$. The object is brought into a room that is kept at a constant temperature T_a. Newton's

law of cooling states that T satisfies the differential equation

$$\frac{dT}{dt} = k(T - T_a)$$

where k is a negative constant.

[6] (a) Suppose that at time $t = 0$ we bring an object whose temperature is

28° C in a room whose temperature is 20° C. Solve for T (note that k

[9] **6.** Solve the differential equation

$$\frac{dy}{dx} = e^{-3y}$$

where $y(0) = 0$.

$$\frac{dy}{dx} = e^{-3y}$$

$$\frac{dy}{e^{-3y}} = dx$$

$$\int e^{3y}\,dy = \int dx$$

$$\frac{1}{3}\cdot e^{3y} = x + C$$

$$e^{3y} = 3x + C_1$$

$(x = 0)$ $$e^{3\cdot 0} = 3\cdot 0 + C_1 \Rightarrow C_1 = 1$$

$$e^{3y} = 3x + 1$$

$$3y = \ln(3x+1)$$

[8] 7. Compute the Taylor polynomial of degree 3 about $x = 0$ for $f(x) = e^x$. Use your result to compute an approximate value of $e^{0.3}$. Compare your approximation with the exact value of $e^{0.3}$.

$$f(x) = e^x \qquad f(0) = 1$$

$$f'(x) = e^x \qquad f'(0) = 1$$

$$f''(x) = e^x \qquad f''(0) = 1$$

$$f'''(x) = e^x \qquad f'''(0) = 1$$

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 =$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

$$P_3(0.3) = 1 + 0.3 + \frac{0.3^2}{2} + \frac{0.3^3}{6} = 1.3495$$

$$e^{0.3} = 1.349858\ldots$$

[2] 9. (a) Write the following system of linear equations in the matrix form:

$$\begin{array}{rrrcr} x_1 & & -x_3 & = & 2 \\ x_1 & +2x_2 & & = & -3 \\ & -x_2 & -x_3 & = & 4 \end{array}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

9. The Gompertz growth model states that if $N(t)$ denotes the population [redacted]

$$\frac{dN}{dt} = rN(\ln K - \ln N)$$

where r and K are positive constants, and $N(t) > 0$ for each t.

[7] (a) Find all equilibria of the Gompertz model and use the eigenvalue method to discuss their stability.

$g(N) = rN(\ln K - \ln N)$ $\quad g'(N) = r(\ln K - \ln N)$

[redacted]

susceptibles, the infectives and the removed class. Write a system of

[10] 11. Suppose a crop yield Y depends on nitrogen (N) and phosphorus (P) concentrations as:

$$Y(N,P) = NPe^{-(N+P)}$$

Find the value of (N, P) that maximizes crop yield.