Use space below if you need extra space to show your work

1. True-False. Circle \T" for true or \F" for false. For this section, you do not need to justify your answers.

[1] (a) **T** or **F** : Every continuous function has an extreme point.

[1] (b) **T** or **F**: The derivative $f^0(x_0)$ exists whe3fl03sd [(f)]FTJ/F28 7.97 continuous tw 7.9701 11 0.284 -1.637 Td [(0

2. Very short answers. Definitions and terminology.

[2] (a) Compute $\lim_{x\to\infty} \frac{7x^7 - 4x^4+5x}{8x^6+5x^3-4}$.

[1] (b) If $u = f(x)$ and $y = g(u)$, state the chain rule for $\frac{dy}{dx}$.

[2] (c) Assume $f(x)$ is differentiable everywhere. Give the definition of the tangent line to f at point $x = a$.

[2] (d) Given differentiable function $f(x)$ and a point x_0. Suppose points $x_1; x_2; \ldots$ are obtained by applying the Newton's method to approximate a zero of f. Write the formula that gives x_{k+1} in terms of x_k.

[2] (e) Let $y(t)$ be the mass of a radioactive material at time t. If the mass is decreasing exponentially with the initial mass 100 grams and half-time 500 years, what is the function $y(t)$?

[2] (f) Write the solution to the differential equation dy

3. Short answers. Show your work.

[3] (a) Find the derivative of the function $f(x) = \ln(\sin(3x^2 + 2x))$.

[3] (b) Find y' if $x^3 + y^3 = 6xy$.

[3] (c) Simplify the expression $\cos(\arctan(x))$.

[3] (d) Find all points not in the domain of $f(x) = \frac{x^2+3x}{x^3+2x^2\ \ 3x}$. For each of these points determine if it is a discontinuity and if so then what kind.

4. Derivatives.

5. Curve sketching. This problem concerns the function $f(x) = x^4 \quad 4x^3$.

[1] (a) Find $f^0(x)$ and $f^{00}(x)$.

[2] (b) Find all critical points of f.

[2] (c) Determine intervals where f is increasing and those where f is decreasing.

[2] (d) Find all local maxima and local minima of f, if any.

[2] (e) Determine intervals where f is concave up and those where f is concave down.

[1] (f) Circle the gure that best approximates the graph of f.

6. Optimization. An open-top box is to have a square base and a volume of 10 m^3. The cost per square metre of material is $5 for the bottom and $2 for the four sides. Let x and y be the width/depth and height of the box, respectively. Let C be the total cost of material required to make the box.

[2] (a) Express C as a function of x and state its domain.

[2] (b) Compute $C'(x)$.

[3] (c) Find the dimensions of the box so that the cost of materials is minimized.

[1] (d) What is this minimum cost?

8. Modelling using di erential equations. The current temperature of a cup of milk sitting on a table is 10 C in a room with temperature of 22 C. The temperature inside the fridge is 4 C. After 20 minutes the temperature of the milk is 12 C. Estimate when the milk was taken out of the fridge.

[2] (a) What is the di erential equation that models the problem?

[2] (b) Write down the general solution for the equation.

[3] (c) Use given information to set up a system that allows you to solve for the unknown time.

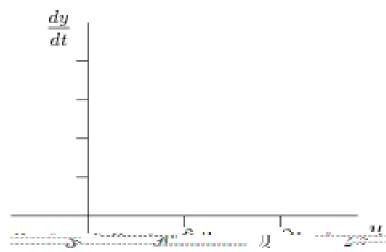
[3] (d) Solve the system and answer the question.

9. Geometry of change. In this problem y is a function of t satisfying the di erential equation

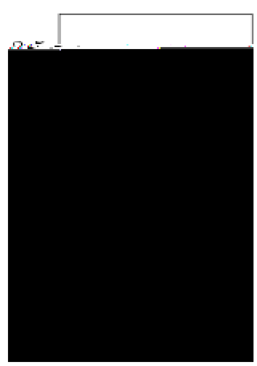
$$\frac{dy}{dt} = y^3 \quad 5y^2 + 6y:$$

[2] (a) What are its steady states?

[2] (b) Graph $\frac{dy}{dt}$ as a function of y on the axes below and plot a state space diagram.



[2] (c) Plot the slope eld in the box below.



[2] (d) What are the stable steady states, if any?

10. Changing angles and related rates. A lighthouse is located on a small island 4km away from the nearest point P on a straight shoreline. The light in the lighthouse makes six revolutions (rotations) per minute. How fast is the beam of light moving along the shoreline when it is 2km from P?

[3] (a) Write the known and unknown rates.

[3] (b) Write the equation relating the known and unknown rates. Start with an equation for the angle .

[4] (c) Solve for the unknown rate and answer the question.