1. Consider the plane in $\mathbb{R}^3$ that goes through the point $(2;1;3)$ and is perpendicular to the vector $(1;1;1)$.

[3] (a) Give a general equation of this plane.

[3] (b) Give a vector equation of this plane.

2. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

[4] (a) Find elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$.

[2] (b) Write A^{-1} as a product of two elementary matrices.

[3] (c) Write A as a product of two elementary matrices.

3. Let $w_1 = (1 ; 1; 0; 1); w_2 = (\quad 1; 1; 0; 0); w_3 = (1 ; 2; 0; 0)$. Let $W = \text{Span}(w_1; w_2; w_3)$.

[5] 4. Let A and B be two square matrices of the same size. Suppose B is singular. Explain why AB is also singular.

5. For each of the linear transformations on $\mathbb{R}^2$ below, determine two linearly independent eigenvectors of the transformation along with their corresponding eigenvalues.

[5] (a) Re ection about the line $y =$ x.

6. Suppose the transition matrix for a Markov process is

	State A	State B
State A	$1 \quad p$	1
State B	p	0

;

where $0 < p < 1$. So, for example, if the system is in state A at time 0 then the probability of being in state B at time 1 is p.

[4] (a) If the system is started in state A at time 0, what is the probability it is in state A at time 2?

[2] (b) The transition matrix is stochastic. Is it regular? Why or why not?

[4] (c) What is the steady-state probability vector?

7. Let $T : R^6 \,!\; R^2$ be a linear operator such that

$$T(x_1; x_2; x_3; x_4; x_5; x_6) = \frac{x_1 + x_2 + x_3}{3}; \frac{x_4 + x_5 + x_6}{3}$$

[3] (a) What is [T], the matrix of T?

[2] (b) What is the rank of [T]?

[2] (c) Give a basis for the range of T.

3tha22 10.9091 Tf 27.931 58.055 P=

9. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$. Let $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[2] (d) Let $v = (1, 1, 1)$. Write v as a linear combination of eigenvectors of B.

[4] (e) What does $B^k v$ converge to as $k \to \infty$?

[4] (f) State the eigenvalues and corresponding eigenvectors of A.