Math 440: Galois Theory

Galois theory is the study of numbers and equations from the perspective of symmetry. Here's an example: take any algebraic equation involving rational numbers and $\sqrt{2}$, like

$$\sqrt{\frac{1}{\sqrt{2}+1}} + \sqrt{\frac{1}{\sqrt{2}+2}} = \sqrt{\frac{1}{\sqrt{2}}}.$$

If you replace every instance of $\sqrt{2}$ by $-\sqrt{2}$, the equation remains true! Check for yourself,

$$\sqrt{\frac{1}{-\sqrt{2}+1}} + \sqrt{\frac{1}{-\sqrt{2}+2}} = -\sqrt{\frac{1}{\sqrt{2}}}.$$