DIAGRAMMING MOBILIZES MATHEMATICS

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Researchers view mathematical diagrams using two diametrically opposed frameworks: one as a visual representation of mathematics, and the other as the virtual that is being actualized as established by Châtelet. This paper presents both views but uses the ideas of Châtelet to create a window into the realm of mathematical thinking and invention by examining how an expert mathematician interacts with diagrams during a research meeting. Findings suggest that a diagram can (1) be a material site for engaging with mathematics, (2) demand a virtual presence within the mathematician, and (3), far from being a static representation, mobilize mathematics.

INTRODUCTION

A few years ago, I entered the PhD program with twenty years of extensive mathematics teaching experience in secondary and post-secondary education behind me. Through my teaching, I have grown to value the role that a diagram plays in conveying mathematical concepts. Having studied literature about visualization as well as gesture research, I am establishing my research framework as grounded in the theories of *virtuality* and *reality* by Châtelet (2000), which establish that the *thought experiment* (the place of intuition, premonition and mathematical abstraction) is the mathematician's *potential* playground, and that the *hinge-horizon* (the place of making sense of objects, their actions and their relationships) must occur as the mathematician *realizes* new mathematics. The key element in Châtelet's framework is the diagram (*actual*), as the written form of gestures (*virtual*), that helps shed light on the creative processes in mathematizing. I find it intriguing that a diagram should hold more meaning than the few lines that are needed to draw it, and that the very act of drawing the diagram and engaging with it can possibly provide keys about one's understanding and invention of mathematics. I am therefore interested in the process of diagramming and the interaction of an expert mathematician with the diagram during mathematizing; and furthermore, how this may lead to the understanding and invention of mathematics. In this work, I briefly summarize the diametrically opposed frameworks developed around diagrams in the research community, and then I report on the preliminary findings that demonstrate that the diagram is more than a visual product on its way to formalization in that it has a voice of its own which mobilizes mathematics.

THE DIAGRAM AS THE VISUAL

The late twentieth century saw the rise of visualization studies in the context of mathematical problem solving at both novice and expert levels. This is perhaps not a surprise, as this was also the era when computers became readily available. The rise of cognitive studies during this time also helped visualization research to gain popularity.

“The term visualization [describes] the process of producing or using geometrical or graphical representations of mathematical concepts, principles or problems, whether hand drawn or computer generated” (Zimmermann & Cunningham, 1991, p. 1), as

I suggest that whether visualization is determined to be a crutch, a tool or an ability makes no difference; the problem is that these views hinder the analysis of the inventive mathematical moments during diagramming, because the mathematical objects and relations that are under consideration are reduced to a static external representation. This view neglects to investigate how the diagram arises and forgets about the hand that draws the diagram and all the experiences that reside in the diagramming person.

THE DIAGRAM AS THE VIRTUAL

In his foreword to Châtelet's *Figuring Space* (2000), Knoespel writes that "in contemporary Greek the verb Διαγραμμα [refers] to writing on a wax note pad in which the mark of a stylus would simultaneously cross over the marks that had been drawn previously. Διαγραμμα in effect embodies a practice of figuring and defiguring" (p. xvi). It is this flexible view of *figuring* and *defiguring* through which Châtelet brings new ideas to the study of gestures, diagrams and mathematics that have influenced recent studies in philosophy, mathematics education and social sciences.

Châtelet (2000) restricts his research to historical, mathematical figureheads such as Oresme, Leibniz, Cauchy and Poisson, and their surviving written manuscripts. Châtelet had no access to sound- or video-recordings of these mathematicians during their research. Instead, he needed to delve deep into the research of these mathematicians and trace their thoughts and actions purely from their writing and their diagramming. In Châtelet's observational journey from mathematical research to the construction of a mathematically sound theoretical framework he was searching for "strategies of cognitive intuition" (Knoespel, 2000, p. xii). The crux of Châtelet's study is his demonstration of how the virtual is evoked in historical mathematical inventions "through diagramming experiments whose sources Châtelet can trace to mobile gestural acts" (Sinclair & de Freitas, 2014, p. 563). It is this trace of the gesture lingering in the invention of new mathematics that is provocative and challenging to the abstract nature that has come to be associated with mathematics. In this way, Châtelet is pointing to the physical nature of mathematics that is necessary for its own invention, and the underlying key ideas are summarized here:

(1) The diagram is never really fixed – it is erased, drawn over, reassembled, or redrawn. In terms of Châtelet, the diagram hovers between virtuality and actuality.

(2) Not only does the mathematician interact with the diagram, the diagram also interacts with the mathematician. This alludes to the diagram having a life on its own.

(3

gestures that the diagram provokes and captures, this virtuality is being actualized. In the words of de fall of 2013. The seminar room is equipped with three adjacent blackboards, each mounted on a vertical sliding track so that a second blackboard behind becomes visible when the front blackboard is shifted up. I attended as an observer and recorder, so that discussions as befitted the research meeting can be considered to have taken place ‘normally’. However, I was also present as semi-participant without talking. My presence in these roles allowed me to make immediate field notes that addressed both the mathematics that developed during the meeting, and observations about diagramming or gesturing that stood out for me while the recoerdi2 . 1 (e) 3 .

Upon conclusion, he walks towards the audience facing them. While he talks about something that is familiar to the other mathematicians in the room, he gestures as if to retrieve this old math (Figure 1a), but does not point at the blackboard where the old math resides. The instant when he connects the just established operation to this old math in speech, his body whips around and points decisively to the just drawn diagram (Figure 1b). I refer to this as the diagram *pulling* on the expert mathematician as if connected by a *tether*. But this is not enough: he then walks back to the blackboard and gestures as if to *grab* the diagram (Figure 1c), and uses the precise terms “di-graph”, “embedded” and “surface” that are evoked through the diagram. Standing now next to the diagram, he assertively connects new to old math in speech, this time pointing to the old math displayed above the new math (Figure 1d).

a	b	c	d	e	f	g

either connects old to new mathematics or elaborates on the new mathematics. But when Morley discusses other researchers' work, or reproduces a theorem on the blackboard or draws diagrams as examples of some mathematical concept, then this pulling of the diagram is not exhibited in either speech or gesture.

I am now in a position to attend to my research questions: *What role does the diagram play in the expert mathematician's practice?* There are three key roles of the diagram that speak to its materiality, immanence and mobilization: (1) Through the pointing, grabbing and touching gestures, it is evident that the diagram is material to the mathematician. (2) The flow of Morley's talk is often interrupted by guttural sounds of "er" denoting hesitation. Precisely at the moments, when Morley physically turns to the diagram and looks at it, he is able to find a word associated with an object depicted in the diagram that allows talk to continue to flow. Together with the tether-like connection described above, this shows that the diagram does not merely exist on the blackboard; it also exists within the mathematician. (3) The diagram is not just the specific mathematical object that it depicts, it often stands for a general class that this object belongs to. Furthermore, the expert mathematician *physically* manipulates the diagram by adding to it, erasing parts or covering up parts with his hand(s) or body, but he also *virtually* it t-0.00h4(t)2.e.004 T ve.004 Tr-12.1(ct)-12(e)3.55 Tf --168231.m04