[illegible]

Karel Proesmans,[1,2,*] Jannik Ehrich,[1] and John Bechhoefer[1]

[1]*De a me f Ph ic , Sim F a e U i e i , B ab , B i i h C umbia, V5A 1S6, Ca ada*

[2]*Ha el U i e i , B-3590 Die e beek, Belgium*

(Received 5 June 2020; accepted 14 August 2020; published 3 September 2020)

We study the thermodynamic cost associated with the erasure of one bit of information over a finite

where $V(x;t)$ is the potential energy landscape. In Eq. (1), we have scaled energy by $k_B T$ and lengths by $x_0 \equiv \sqrt{\mathrm{Var}(x)}$, the variance of the equilibrium distribution for the potential $V(x;0) \equiv V_0(x)$. Time is scaled by x_0^2/D, with D the diffusion coefficient. This description applies to a broad class of systems, including colloidal particles trapped in a potential [5–8] and superconducting fluxes [9]. In such systems, the microscopic state $x(t)$ is coarse grained to two (or more) macrostates that encode information [1].

Building on ideas from stochastic thermodynamics [14,15] and optimal-transport theory [28,29], one can calculate the minimum average work to go from an initial microscopic equilibrium distribution $p_0(x)$ to a final microscopic distribution $p_\tau(x)$ over a time interval of length τ for protocols having the same start and end point, with $V(x;0) = V(x;\tau) \equiv V_0(x)$. Assuming full control over the potential $V(x;t)$, one finds [25,26,30,31]

$$W = \underbrace{\int_{-\infty}^{\infty} dx p_\tau(x) \ln \frac{p_\tau(x)}{p_0(x)}}_{\Delta\mathcal{F}} + \underbrace{\frac{1}{\tau}\int_0^1 dy [f_0^{-1}(y) - f_\tau^{-1}(y)]^2}_{\Delta_i S}; \tag{2}$$

where $f_{0=\tau}(x) = \int_{-\infty}^{x} dx' p_{0=\tau}(x')$ are the associated cumulative distributions and f^{-1} their inverses. The quantity $\Delta\mathcal{F}$ is the change in nonequilibrium free energy arising solely because the probability density is transformed from $p_0(x)$ to $p_\tau(x)$, and $\Delta_i S$

corresponding densities $p(x;t)$ for $\tau = 0.2$. Once the optimal $\Gamma(x)$ is determined, the intermediate probability distribution $p(x;t)$ can be computed, which gives the intermediate control protocol $V(x;t)$ by inverting the Fokker-Planck equation [Eq. (1)]. See Ref. [27], Sec. II for full details. The protocol has jump discontinuities when passing from $V_0(x)$ (black curve) at $t = 0^-$ to $V(x; t = 0^+)$ (red curve) and similarly in passing from $V(x; t = 1^-)$ to $V_1(x) = V_0(x)$. At $t = \tau$, we add a $\delta(0)$ barrier that keeps probability from leaking back into the right well for $t > \tau$. Notice that no work is done for $t > \tau$. The probability that is trapped in the left well then relaxes to local equilibrium, after which the barrier may be removed. See the bottom plots.

We then numerically calculate the upper and lower bounds [Eqs. (7) and (8)] and W_{min} and $W_{min;leq}$ for full erasure. Figure 3 shows that the upper and lower bounds are satisfied. We also note that $W_{min} \approx W_{min;leq}$ in the slow-driving limit and that W

[17] M. V. Bonança and S. Deffner, Optimal driving of isothermal processes close to equilibrium, J. Chem. Phys. **140**, 244119 (2014).
[18] D. A. Sivak and G. E. Crooks, Thermodynamic Metrics and Optimal Paths, Phys. Rev. Lett. **10**1, 190602 (2012).

Supplementary Material for Finite-time Landauer Principle

Karel Proesmans,[1,2] Jannik Ehrich,[1] and John Bechhoefer[1]

[1]

as the system ends up in local equilibrium in the slow-driving limit. A straightforward calculation, using Eq. (3) from the main text (see also Ref. [4]) leads to

$$\frac{W_{\mathrm{min}}}{k_{\mathrm{B}}T} \quad \ln 2 = \frac{\mathrm{Var}(x)}{D} : \tag{S12}$$

Boyd et al. [8]: The probability distribution is always kept in local equilibrium,

$$p(x;t) = \begin{cases} [1 \quad b(t)]p_0(x) & x < 0 \\ b(t)p_0(x) & x > 0 ; \end{cases} \tag{S13}$$

where $b(t) = p(x > 0; t)$ is the probability for the bit to be in state 1 at time t. The authors show that the required work to erase a bit under such a constraint is given by

$$\frac{W}{k_{\mathrm{B}}T} \quad \ln 2 \quad \ln \quad (1 \quad) \ln(1 \quad) = f_1[p_0(x)] \, f_2[b(t)] ; \tag{S14}$$

with [?]

$$f_1[p_0(x)] = \frac{2}{D} \int_0^{1} \mathrm{d}x \int_0^{x} \mathrm{d}x^{0} \int_{1}^{x^{0}} \mathrm{d}x^{00} \, \frac{p(x)p(x^{00})}{p(x^{0})}$$

$$f_2[b(t)] = \int_0 \mathrm{d}$$