(1) The output is updated after an observation with a delay time t_d.

(2) The observation is made via a camera exposure of duration t_c.

(3) The update is applied for a time t_s, which is also the

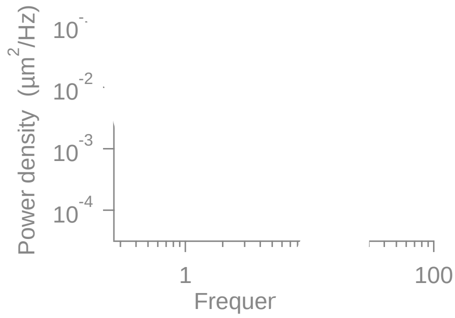
In Eq. (6), we omit the terms $2_{n-1}Ø_n = -2\,Ø_{n-1}\,Ø_n = 0$ (no overlap). We can evaluate

$$
\begin{aligned}
{}_{n-1}Ø_{n-1} &= \frac{1}{t_c}\left\langle \int_0^{t_s} {}^{(v)}(t')\,dt' \int_0^{t_c} dt \int_0^{t} {}^{(v)}(t'')\,dt'' \right\rangle \\
&= \frac{1}{t_c}\int_0^{t_c} dt \int_0^{t} dt'' \int_0^{t_s} \left\langle {}^{(v)}(t')\, {}^{(v)}(t'')\right\rangle dt' \\
&= \frac{2D}{t_c}\int_0^{t_c} dt \int_0^{t} dt'' \int_0^{t_s} (t' - t'')\,dt' \\
&= \frac{2D}{t_c}\int_0^{t_c} dt \int_0^{t} (1)\,dt'' \\
&= \frac{2D}{t_c}\int_0^{t_c} t\,dt \\
&= Dt_c, \qquad (7)
\end{aligned}
$$

where we have shifted the domain of integration for all the integrals by $(n-1)t_s - \frac{1}{2}t_c$, for clarity. A similar calculation by Cohen [9] gives $(Ø_n)^2 = \frac{2}{3}Dt_c$. Putting these results together, we have

$$
\begin{aligned}
(\overline{x}_n)^2 &= 2Dt_s + 2\left(\tfrac{2}{3}Dt_c\right) - 2(Dt_c) \\
&= 2D\left(t_s - \tfrac{1}{3}t_c\right), \qquad (8)
\end{aligned}
$$

which gives the finite-exposure-time correction to the usual


Power density (µm²/Hz)
10⁻
10⁻²
10⁻³
10⁻⁴
1
100
Frequer

As promised, the power spectrum and related results are just slightly more complicated than for integer delay. Indeed, it is generally true that fractional delays merely change coefÞcients in a discrete dynamical system [13].

III. VIRTUAL POTENTIALS AND THERMODYNAMICS

Can virtual potentials be used for thermodynamic calculations? In this section, we investigate the accuracy of ÒnaiveÓ calculations of the work done by a changing potential, following ideas of stochastic thermodynamics [17Ð19]. As an example calculation, we calculate the mean work required to vary the stiffness of a virtual harmonic potential in a Þnite time. We will Þnd that estimates of work agree, to $O( )$, with those of a true potential.

A. Harmonic potential with varying force constant

To explore the work done by a virtual potential, we consider a time-dependent potential $U(x,t)$. We start with the case of a quadratic virtual potential with a feedback gain $_n$ that is increased in constant steps from $_i$ at $t = 0$ to $_f$ at $t = =$ $N t_s$. Recall that $_n$ corresponds to a force $F_n = Š k_n x_n$, but k_n only approximates the force constant of a harmonic potential.

To begin, we recall the calculation of the average work done in the continuous case, where the true force constant $k^{(c)}(t)$ varies from $k_i^{(c)}$ to $k_f^{(c)}$. We further assume that the variation is done slowly enough ( ) that, at each moment, the system is a929 Tm -0.0058 T3que31.j 6.9hium.m=ŠWNtv T90.1 F1 1 Tf 6.9738 27973 Tc 604-317.5(8Tc (t)T UF1 1ET0.2G

Fig. 6(a) we see the approach to the value (horizontal black line) calculated from Eq.